Unit IV: Vectors and Three-Dimensional Geometry (Advanced)

General Instructions

1. Total Questions: **20**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Read each question carefully before answering.
5. Choose the most appropriate answer from the given options.
6. Use of calculator or electronic devices is strictly prohibited.

1. Find the value of λ such that the four points $A(3,2,1)$, $B(4,\lambda,5)$, $C(4,2,-2)$, and $D(6,5,-1)$ are coplanar.

2. Find the equation of the line passing through $(1,2,-4)$ and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

3. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, find the value of x.

4. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

5. Find the coordinates of the point where the line joining $A(3,4,1)$ and $B(5,1,6)$ crosses the XY-plane.

6. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors such that $\vec{b} \times \vec{c} = \vec{a}$, $\vec{c} \times \vec{a} = \vec{b}$ and $\vec{a} \times \vec{b} = \vec{c}$, find the value of $|\vec{a}| + |\vec{b}| + |\vec{c}|$.

7. Find the reflection (image) of the point $(1,2,3)$ in the line $\vec{r} = 6\hat{i} + 7\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$.

8. Determine the angle between the diagonals of a cube.

9. Find the area of a triangle whose vertices are the mid-points of the sides of the triangle with vertices $(0,0,0)$, $(2,0,0)$, and $(0,2,0)$.

10. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew lines.

11. Find the vector equation of the line passing through the point $(2,3,2)$ and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.

12. If the volume of a tetrahedron whose vertices are $(1,1,1)$, $(3,\lambda,1)$, $(1,2,1)$, and $(2,2,5)$ is 2 cubic units, find the value of λ.

13. A line passes through $(2,-1,3)$ and is perpendicular to the lines $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} + 2\hat{j} + \hat{k})$. Find its equation.

14. Find the projection of the line segment joining $(2,1,3)$ and $(5,1,-1)$ on the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{-6}$.

15. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, find $|\vec{x}|$.

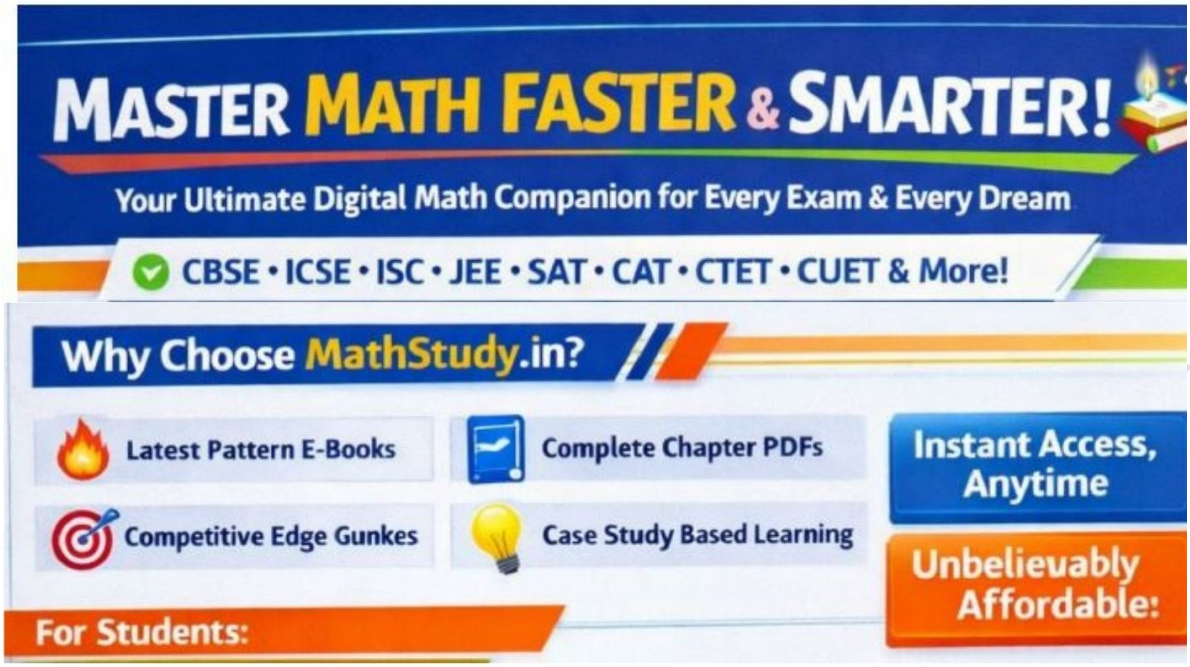
16. Show that the vectors $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right-angled triangle.

17. Find the position vector of the point which divides the join of $P(2\vec{a} + \vec{b})$ and $Q(\vec{a} - 3\vec{b})$ externally in the ratio $1:2$.

18. Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to $(1,-2,-2)$ and $(0,2,1)$.

19. Calculate the perpendicular distance of the point $P(1,0,0)$ from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$.

20. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ from the point $(1,2,3)$.



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