Unit IV: Vectors and Three-Dimensional Geometry (Advanced)

SOLUTIONS

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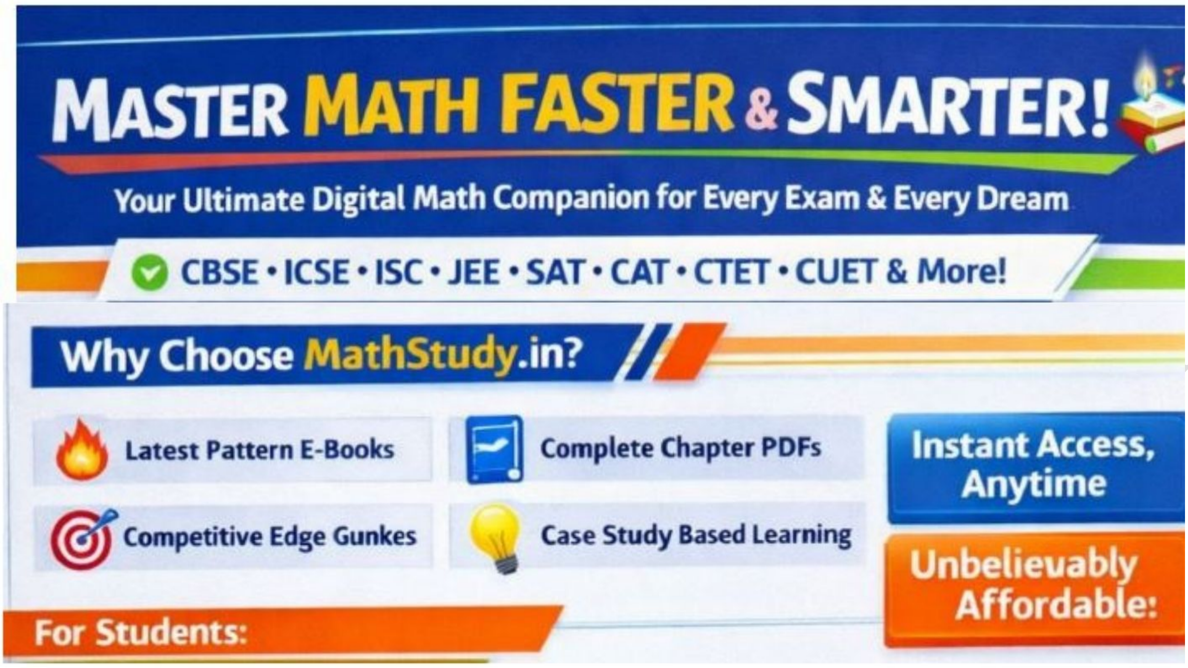

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Solutions

1. **Solution:** Vectors $\vec{AB} = (1, \lambda - 2, 4)$, $\vec{AC} = (1, 0, -3)$, $\vec{AD} = (3, 3, -2)$. Scalar Triple Product $[AB, AC, AD] = 0$. Result: $\lambda = 5$.

2. **Solution:** Direction of required line is the cross product of $(3, -16, 7)$ and $(3, 8, -5)$, which is $(24, 36, 72)$ or $(2, 3, 6)$. Result: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$.

3. **Solution:** Coplanar means $[\vec{a}, \vec{b}, \vec{c}] = 0$. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$. Result: $x = -4$.

4. **Solution:** Lines are $\vec{r_1} = (1, 2, 3) + \lambda(2, 3, 4)$ and $\vec{r_2} = (2, 4, 5) + \mu(3, 4, 5)$. $\vec{b_1} \times \vec{b_2} = (-1, 2, -1)$. $SD = \frac{|(1,2,2)\cdot(-1,2,-1)|}{\sqrt{1+4+1}} = \frac{1}{\sqrt{6}}$.

5. **Solution:** XY-plane means $z = 0$. Line: $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$. Set $z = 0 \Rightarrow \frac{x-3}{2} = -1/5$ and $\frac{y-4}{-3} = -1/5$. Result: $(13/5, 23/5, 0)$.

6. **Solution:** $|\vec{a}| = |\vec{b}||\vec{c}| \sin 90°$ etc. This implies $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$. Result: 3.

7. **Solution:** Foot of perpendicular is $(3, 5, 9)$. Image = 2(Foot) − Point. Result: $(5, 8, 15)$.

8. **Solution:** Diagonals: $(1, 1, 1)$ and $(1, 1, -1)$. $\cos\theta = \frac{1+1-1}{\sqrt{3}\sqrt{3}} = 1/3$. Result: $\cos^{-1}(1/3)$.

9. **Solution:** Mid-points: $(1, 0, 0), (0, 1, 0), (1, 1, 0)$. This is a right triangle in the XY-plane. Result: $1/2$ sq. units.

10. **Solution:** Check if they intersect by solving $\vec{a_1} + \lambda\vec{b_1} = \vec{a_2} + \mu\vec{b_2}$. The system of equations is inconsistent.

11. **Solution:** Parallel line: $\vec{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Distance between parallel lines calculation results in $\frac{\sqrt{580}}{7}$.

12. **Solution:** Volume $= \frac{1}{6}|[\vec{AB}, \vec{AC}, \vec{AD}]| = 2$. Vectors $(2, \lambda - 1, 0), (0, 1, 0), (1, 1, 4)$. Result: $\lambda = 4$ or -2.

13. **Solution:** Direction vector: $(2, 1, -3) \times (1, 2, 1) = (7, -5, 3)$. Result: $\frac{x-2}{7} = \frac{y+1}{-5} = \frac{z-3}{3}$.

14. **Solution:** Vector $\vec{PQ} = (3, 0, -4)$. Line direction $\vec{n} = (2/7, 3/7, -6/7)$. Projection $= |\vec{PQ} \cdot \vec{n}| = |6/7 + 0 + 24/7| = 30/7$.

15. **Solution:** $|\vec{x}|^2 - |\vec{a}|^2 = 8 \Rightarrow |\vec{x}|^2 - 1 = 8 \Rightarrow |\vec{x}| = 3$.

16. **Solution:** $\vec{a} + \vec{c} = (5, -1, -3)$. Not a closed loop. Check $\vec{a} + \vec{b} + \vec{c} \neq 0$. Actually, check dot products. $\vec{a} \cdot \vec{c} = 6 - 2 - 4 = 0$. Since $\vec{a} \perp \vec{c}$, it is a right-angled triangle.

17. **Solution:** Section formula: $\frac{1(\vec{a}-3\vec{b})-2(2\vec{a}+\vec{b})}{1-2} = \frac{-3\vec{a}-5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$.

18. **Solution:** Cross product of $(1, -2, -2)$ and $(0, 2, 1)$ is $(2, -1, 2)$. Magnitude is 3. Result: $(2/3, -1/3, 2/3)$.

19. **Solution:** Point on line $M(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$. $\vec{PM} \cdot (2, -3, 8) = 0 \Rightarrow 4\lambda + 9\lambda + 3 + 64\lambda - 80 = 0$. $\lambda = 1$. Distance $PM = \sqrt{2^2 + (-4)^2 + (-2)^2} = \sqrt{24} = 2\sqrt{6}$.

20. **Solution:** Point $P(3\lambda-2, 2\lambda-1, 2\lambda+3)$. Distance from $(1, 2, 3)$ is $\sqrt{(3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2} = \sqrt{18}$. Result: $\lambda = 1 \Rightarrow (1, 1, 5)$ or $\lambda = 9/17$ (approx).