Unit I: Relations, Functions, and Inverse Trigonometry

SOLUTIONS

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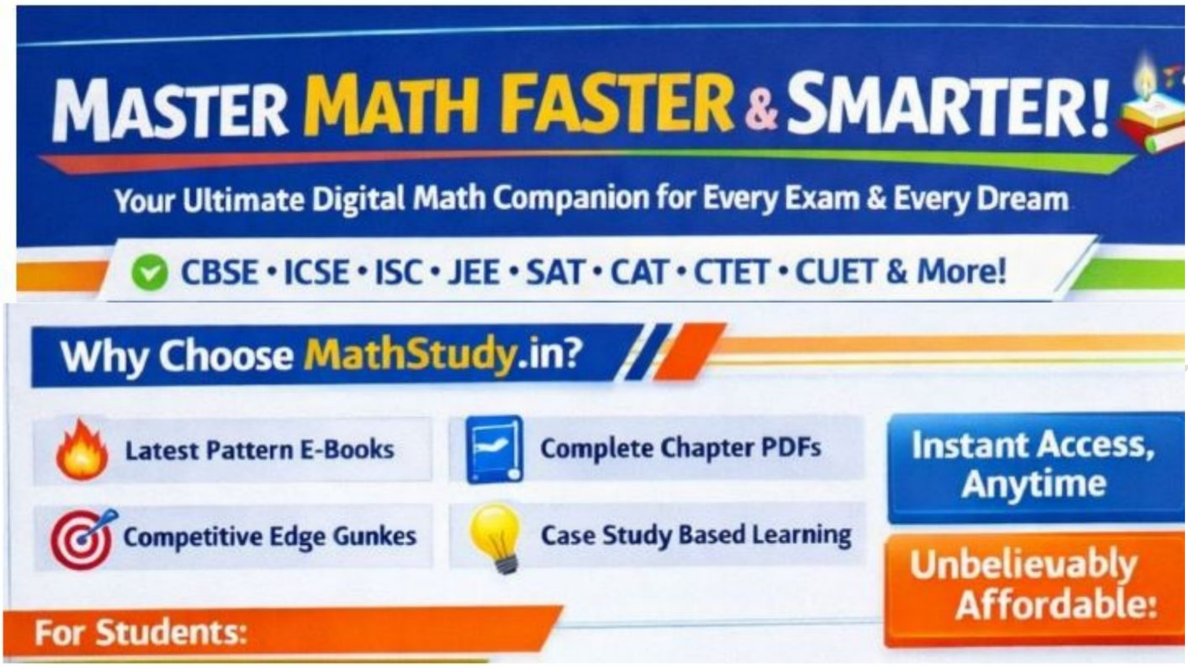

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Solutions

1. **Solution:** (i) Reflexive: $x^2 + x^2 = 2x^2$ (always even). (ii) Symmetric: $x^2 + y^2$ even $\Rightarrow y^2 + x^2$ even. (iii) Transitive: If $x^2 + y^2$ and $y^2 + z^2$ are even, their sum $(x^2 + z^2) + 2y^2$ is even, hence $x^2 + z^2$ is even. **Result: Equivalence Relation.**

2. **Solution:** Formula for onto functions $n \to 2$ is $2^n - 2$. Here $2^6 - 2 = 64 - 2 = 62$. **Result: 62.**

3. **Solution:** Let $y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$. **Result:** $f^{-1}(x) = \frac{3x-2}{x-1}$.

4. **Solution:** Let $a = -1, b = 0, c = 1$. $1 + ab = 1 + 0 = 1 > 0$ (True). $1 + bc = 1 + 0 = 1 > 0$ (True). $1 + ac = 1 - 1 = 0 \not> 0$. **Result: Not Transitive.**

5. **Solution:** Requires $\frac{5x-x^2}{4} \geq 1 \Rightarrow x^2 - 5x + 4 \leq 0 \Rightarrow (x-1)(x-4) \leq 0$. **Result:** $x \in [1, 4]$.

6. **Solution:** For f to be one-one, $f'(x) \geq 0$ for all x. $3x^2 + 2(a+2)x + 3a \geq 0$. Discriminant $D \leq 0 \Rightarrow 4(a+2)^2 - 4(3)(3a) \leq 0 \Rightarrow a^2 - 5a + 4 \leq 0$. **Result:** $a \in [1, 4]$.

7. **Solution:** $\frac{2x+3x}{1-6x^2} = 1 \Rightarrow 6x^2 + 5x - 1 = 0 \Rightarrow (6x-1)(x+1) = 0$. $x = 1/6$ (Valid), $x = -1$ (Invalid as it makes LHS negative). **Result:** $x = 1/6$.

8. **Solution:** $\cos(\cos^{-1} x + \cos^{-1} x + \sin^{-1} x) = \cos(\cos^{-1} x + \pi/2) = -\sin(\cos^{-1} x) = -\sqrt{1-x^2}$. At $x = 1/5$, value is $-\sqrt{1 - 1/25} = -\sqrt{24}/5$. **Result:** $-2\sqrt{6}/5$.

9. **Solution:** $f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4/4^x}{4/4^x+2} = \frac{4^x}{4^x+2} + \frac{2}{2+4^x} = 1$. There are 49 pairs totaling 49, plus $f(1/2) = 1/2$. **Result: 49.5.**

10. **Solution:** (i) Periodicity of cosine makes it many-one. (ii) Range is $[-1, 1]$, codomain is $\mathbb{R}$, so not onto.

11. **Solution:** $\sin^{-1}(\sin(\pi - \pi/3)) + \cos^{-1}(\cos(2\pi - 2\pi/3)) = \pi/3 + 2\pi/3 = \pi$. **Result:** π.

12. **Solution:** $n(S \times S) = 16$. 4 pairs (x, x) must be included. Remaining 12 pairs can either be in or out. **Result:** $2^{12} = 4096$.

13. **Solution:** Max value of $\sin^{-1}$ is $\pi/2$. Sum $3\pi/2 \Rightarrow x = y = z = 1$. $1 + 1 + 1 - 9/(3) = 3 - 3 = 0$. **Result: 0.**

14. **Solution:** $x^2 + x$ must be ≥ 0. $\sqrt{x^2 + x} \in [0, \infty)$. $\tan^{-1}[0, \infty) = [0, \pi/2)$. **Result:** $[0, \pi/2)$.

15. **Solution:** (i) $1R2$ but $2 \not R 1$ (Not symmetric). (ii) $|a| \leq b, |b| \leq c \Rightarrow |a| \leq b \leq |b| \leq c \Rightarrow |a| \leq c$ (Transitive). **Result: Transitive but not Symmetric.**

16. **Solution:** Let $\cos\theta = \sqrt{5}/3$. $\tan(\theta/2) = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$. **Result:** $\frac{3-\sqrt{5}}{2}$.

17. **Solution:** Standard property holds for $|x| \leq 1$. **Result:** $x \in [-1, 1]$.

18. **Solution:** $\tan^{-1}(n+1) - \tan^{-1} n$. Telescoping sum: $\lim_{N\to\infty}(\tan^{-1}(N+1) - \tan^{-1} 1) = \pi/2 - \pi/4$. **Result:** $\pi/4$.

19. **Solution:** $f(1)$ has $(n-1)$ choices. Other $(n-1)$ elements have n choices each. **Result:** $(n-1)n^{n-1}$.

20. **Solution:** $\tan^{-1}\left(\frac{x/y-(x-y)/(x+y)}{1+(x/y\cdot\frac{x-y}{x+y})}\right) = \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right) = \tan^{-1}(1)$. **Result:** $\pi/4$.