CUET Mathematics Test - Set 27

Unit IV: Probability Distributions (Intermediate to Advanced)

General Instructions

1. Total Questions: **15**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Each question carries **5 marks**.
5. For each correct answer: **+5 marks**.
6. For each incorrect answer: **-1 mark**.
7. No negative marking for unanswered questions.
8. Use of calculator or electronic devices is strictly prohibited.
9. Choose the most appropriate answer from the given options.

1. A random variable X has a mean of 10 and a variance of 25. For what value of a is the expectation $E[(X-a)^2]$ minimized?
   (A) 0
   (B) 5
   (C) 10
   (D) 25

2. If X follows a Binomial distribution with parameters n and p, and $E(X) = 20$, $Var(X) = 16$, then the value of p is:
   (A) 0.2
   (B) 0.4
   (C) 0.8
   (D) 0.5

3. In a Poisson distribution, if $P(X = k)$ is maximum when $k = 3$ and $k = 4$, then the mean λ of the distribution is:
   (A) 3
   (B) 3.5
   (C) 4
   (D) 12

4. If X is a Poisson random variable such that $P(X \geq 1) = 1 - e^{-3}$, then $Var(X)$ is:
   (A) 3
   (B) 9
   (C) $\sqrt{3}$
   (D) e^{-3}

5. For a normal distribution, the probability of a value falling within two standard deviations from the mean (i.e., $\mu - 2\sigma$ to $\mu + 2\sigma$) is approximately:
   (A) 0.682
   (B) 0.954
   (C) 0.997
   (D) 0.500

6. Let X be a random variable with probability density function $f(x) = kx^2$ for $0 < x < 3$ and 0 otherwise. The value of k is:
   (A) 1/3
   (B) 1/9
   (C) 1/27
   (D) 1/10

7. If the probability of a defective bolt is 0.1, the mean and standard deviation for the number of defective bolts in a total of 400 bolts are:
   (A) 40, 6
   (B) 40, 36
   (C) 40, 20
   (D) 40, 10

8. If $E(X) = 1$ and $Var(X) = 5$, then $E[(2 + X)^2]$ is:
   (A) 9
   (B) 14
   (C) 10
   (D) 15

9. In a Binomial distribution $B(n, p)$, the mean is 6 and the variance is 2. The distribution is:
   (A) Symmetrical
   (B) Positively skewed
   (C) Negatively skewed
   (D) None of these

10. A fair die is tossed 180 times. The expected number of times a '6' will appear is:
   (A) 20
   (B) 30
   (C) 60
   (D) 10

11. If X is a Poisson variable such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$, then the mean of X is:
   (A) 1
   (B) 2
   (C) 3
   (D) 4

12. The points of inflection of a normal curve with mean μ and standard deviation σ occur at:
   (A) $x = \mu \pm \sigma$
   (B) $x = \mu \pm 2\sigma$
   (C) $x = \mu \pm 0.67\sigma$
   (D) $x = \mu$

13. For a random variable X, $E(X) = 2$ and $E(X^2) = 8$. The value of $E[(X - 2)^2]$ is:
   (A) 4
   (B) 8
   (C) 2
   (D) 0

14. If 10% of the items produced by a machine are defective, the probability that out of 5 items chosen at random, at most one is defective is:
   (A) $(0.9)^5 + 0.5(0.9)^4$
   (B) $(0.9)^5 + 0.45(0.9)^4$
   (C) $(0.9)^5 + (0.9)^4$
   (D) $0.5(0.9)^4$

15. If X is a random variable such that $E(X^r) = 2^r$ for $r = 1, 2, 3 \ldots$, then $Var(X)$ is:
   (A) 0
   (B) 1
   (C) 2
   (D) 4

Solutions

1. **Solution:** The expression $E[(X-a)^2]$ is minimized when $a = E(X)$. Given $E(X) = 10$, $a = 10$. **Correct Option: (C)**

2. **Solution:** $np = 20$ and $npq = 16$. $q = 16/20 = 0.8$. Thus $p = 1 - 0.8 = 0.2$. **Correct Option: (A)**

3. **Solution:** In a Poisson distribution, if the mode occurs at two points $k-1$ and k, then the mean $\lambda = k$. Here $k = 4$, so $\lambda = 4$. **Correct Option: (C)**

4. **Solution:** $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-\lambda}$. Given $1 - e^{-\lambda} = 1 - e^{-3} \implies \lambda = 3$. In Poisson, $Var(X) = \lambda = 3$. **Correct Option: (A)**

5. **Solution:** By the Empirical Rule for Normal Distribution, approx 95.4% of data falls within $\pm 2\sigma$. **Correct Option: (B)**

6. **Solution:** Total probability $\int_0^3 kx^2 dx = 1 \implies [k\frac{x^3}{3}]_0^3 = 1 \implies 9k = 1 \implies k = 1/9$. **Correct Option: (B)**

7. **Solution:** $n = 400, p = 0.1$. Mean $np = 40$. $Var = npq = 400(0.1)(0.9) = 36$. $SD = \sqrt{36} = 6$. **Correct Option: (A)**

8. **Solution:** $E[(2+X)^2] = E[4 + 4X + X^2] = 4 + 4E(X) + E(X^2)$. Since $Var(X) = E(X^2) - [E(X)]^2$, $5 = E(X^2) - 1^2 \implies E(X^2) = 6$. Total $= 4 + 4(1) + 6 = 14$. **Correct Option: (B)**

9. **Solution:** $np = 6, npq = 2 \implies q = 1/3, p = 2/3$. Since $p > 0.5$, the distribution is negatively skewed. **Correct Option: (C)**

10. **Solution:** This is a Binomial distribution problem with $n = 180$ and $p = 1/6$. $E(X) = np = 180 \times (1/6) = 30$. **Correct Option: (B)**

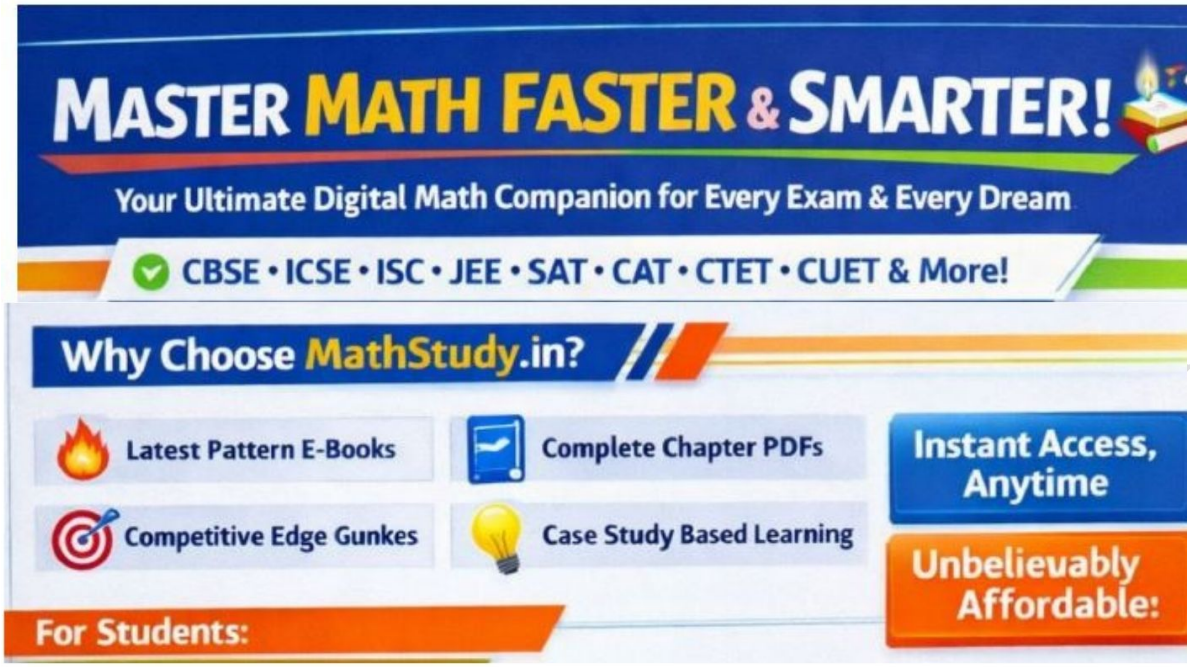
11. **Solution:** Substitute $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$. $\frac{\lambda^2}{2} = 9\frac{\lambda^4}{24} + 90\frac{\lambda^6}{720}$. Simplifying: $\frac{\lambda^2}{2} = \frac{3\lambda^4}{8} + \frac{\lambda^6}{8}$. Let $\lambda^2 = y$. Solve $y^2 + 3y - 4 = 0 \implies (y+4)(y-1) = 0 \implies y = 1 \implies \lambda = 1$. **Correct Option: (A)**

12. **Solution:** In a Normal curve $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$, the second derivative $f''(x) = 0$ at $x = \mu \pm \sigma$. **Correct Option: (A)**

13. **Solution:** $E[(X-2)^2] = E[X^2 - 4X + 4] = E(X^2) - 4E(X) + 4 = 8 - 4(2) + 4 = 4$. **Correct Option: (A)**

14. **Solution:** $p = 0.1, q = 0.9, n = 5$. $P(X \leq 1) = \binom{5}{0}(0.9)^5 + \binom{5}{1}(0.1)(0.9)^4 = (0.9)^5 + 5(0.1)(0.9)^4 = (0.9)^5 + 0.5(0.9)^4$. **Correct Option: (A)**

15. **Solution:** $E(X) = 2^1 = 2$. $E(X^2) = 2^2 = 4$. $Var(X) = E(X^2) - [E(X)]^2 = 4 - (2)^2 = 0$. **Correct Option: (A)**



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