

Unit II: Algebra - Matrices

General Instructions

1. Total Questions: **20**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Read each question carefully before answering.
5. Choose the most appropriate answer from the given options.
6. Use of calculator or electronic devices is strictly prohibited.

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1. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Prove by induction that $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$ for all $n \in \mathbb{N}$.
2. If A and B are square matrices of the same order such that $AB = A$ and $BA = B$, show that $A^2 = A$ and $B^2 = B$.
3. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the condition $A^T A = I$.
4. Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.
5. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $A^n = \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$.
6. Let A be a square matrix. If $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.
7. If $A = \begin{bmatrix} 0 & -\tan(\theta/2) \\ \tan(\theta/2) & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A) \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$.
8. Find a 2×2 matrix B such that $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} B = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$.
9. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then show that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$.
10. If A is a skew-symmetric matrix of order n and X is an $n \times 1$ column matrix, prove that $X^T A X = O$.
11. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. Find a matrix B such that $A + B = A^2$.
12. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, find $f(A)$ where $f(x) = x^2 - 2x - 3$.
13. Prove that if A and B are symmetric matrices of the same order, then AB is symmetric if and only if A and B commute.
14. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, find A^n .
15. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $AB = BA$, find the relationship between a, b, c , and d .
16. If A is a square matrix such that $A^2 = A$, prove that $(I + A)^n = I + (2^n - 1)A$ is false, and find the correct expansion.
17. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, find α such that $A^2 = B$.
18. Prove that the product of two upper triangular matrices is an upper triangular matrix.

19. For $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, find A^n in terms of A .

20. Show that the matrix $B^T A B$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

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