

Unit II: Algebra - Matrices

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## Solutions

- Solution:** Base case  $n = 1$  holds. Assume  $A^k = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ . Then  $A^{k+1} = A^k \cdot A = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k+1 & 1 \end{bmatrix}$ .
- Solution:**  $A^2 = A \cdot A = (AB)A = A(BA) = AB = A$ . Similarly for  $B^2 = B$ .
- Solution:**  $A^T A = I$  gives equations:  $2x^2 = 1$ ,  $6y^2 = 1$ ,  $3z^2 = 1$ . **Result:**  $x = \pm \frac{1}{\sqrt{2}}$ ,  $y = \pm \frac{1}{\sqrt{6}}$ ,  $z = \pm \frac{1}{\sqrt{3}}$ .
- Solution:**  $A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$ . Let  $P = \frac{1}{2}(A+A^T)$  (Symmetric) and  $Q = \frac{1}{2}(A-A^T)$  (Skew-symmetric).
- Solution:** Proved by Induction.  $A^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$ , which matches  $n = 2$  in the formula.
- Solution:**  $(A - I)^3 + (A + I)^3 - 7A = (A^3 - 3A^2 + 3A - I) + (A^3 + 3A^2 + 3A + I) - 7A = 2A^3 + 6A - 7A = A$ . Since  $A^2 = I$ ,  $A^3 = A$ . Thus  $2A + 6A - 7A = A$ . **Result:**  $A$ .
- Solution:** Let  $t = \tan(\theta/2)$ .  $I - A = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$ . Multiply by the rotation matrix using half-angle formulas for  $\cos \theta$  and  $\sin \theta$ .
- Solution:**  $B = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ .
- Solution:** Use the identity  $\cos A \cos B - \sin A \sin B = \cos(A + B)$  and induction.
- Solution:** Let  $k = X^T A X$ .  $k$  is a  $1 \times 1$  matrix (scalar).  $k = k^T = (X^T A X)^T = X^T A^T X = X^T (-A) X = -k$ . Thus  $2k = 0 \Rightarrow k = 0$ .
- Solution:**  $A^2 = \begin{bmatrix} 1 & -6 \\ 18 & 13 \end{bmatrix}$ .  $B = A^2 - A = \begin{bmatrix} -1 & -5 \\ 15 & 9 \end{bmatrix}$ .
- Solution:**  $A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ ,  $2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ ,  $3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ .  $f(A) = A^2 - 2A - 3I = O$ .
- Solution:**  $(AB)^T = B^T A^T = BA$ . For  $AB$  to be symmetric,  $(AB)^T = AB \Rightarrow BA = AB$ .
- Solution:**  $A^n = \begin{bmatrix} a^n & b(a^{n-1} + a^{n-2} \dots + 1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^n & b \frac{a^n - 1}{a - 1} \\ 0 & 1 \end{bmatrix}$  for  $a \neq 1$ .
- Solution:** Equating  $AB = BA$  yields  $c = 3b$  and  $d = a + \frac{1}{3}(4c - 3b)$ ? Simplified:  $3b = 2c$  and  $a + 2c = d + 3b$ .
- Solution:** Expansion is  $(I + A)^n = I + \binom{n}{1}A + \binom{n}{2}A^2 \dots = I + A(\binom{n}{1} + \dots) = I + (2^n - 1)A$ . (Wait, the question asked to prove it false or find expansion; it is actually true for idempotent matrices).
- Solution:**  $A^2 = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$ .  $\alpha^2 = 1 \Rightarrow \alpha = \pm 1$ .  $\alpha + 1 = 2 \Rightarrow \alpha = 1$ . **Result:**  $\alpha = 1$ .
- Solution:** Let  $A, B$  be upper triangular.  $(AB)_{ij} = \sum a_{ik} b_{kj}$ . If  $i > j$ , all terms  $a_{ik} b_{kj}$  are zero because either  $i > k$  or  $k > j$ .

19. **Solution:**  $A^2 = 3A, A^3 = 9A$ . **Result:**  $A^n = 3^{n-1}A$ .

20. **Solution:**  $(B^T AB)^T = B^T A^T (B^T)^T = B^T A^T B$ . If  $A^T = A$ , result is  $B^T AB$  (Symmetric).  
If  $A^T = -A$ , result is  $-B^T AB$  (Skew-symmetric).

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