Chapter: Linear Programming (Optimization and Feasibility)

SOLUTIONS

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Solutions

1. **Solution:** Corner points are $(0, 0), (4, 0), (0, 4)$. $Z(0, 0) = 0, Z(4, 0) = 12, Z(0, 4) = 16$. Max value = 16.
2. **Solution:** Intersection of $x + 2y = 10$ and $3x + y = 15$ gives $(4, 3)$. Axes intersections give $(0, 0), (5, 0), (0, 5)$. Corner points: $(0, 0), (5, 0), (4, 3), (0, 5)$.
3. **Solution:** $Z(0, 4) = Z(2, 3) \Rightarrow 4b = 2a + 3b \Rightarrow b = 2a \Rightarrow a/b = 1/2$. Ratio $a : b = 1 : 2$.
4. **Solution:** Intersection of $2x+y = 8$ and $x+2y = 10$ is $(2, 4)$. Corner points: $(0, 8), (2, 4), (10, 0)$. $Z(0, 8) = 40, Z(2, 4) = 4 + 20 = 24, Z(10, 0) = 20$. Min value = 20.
5. **Solution:** $x + y \le 1$ and $x + y \ge 2$ imply $2 \le x + y \le 1$, which is impossible. Feasible region is ϕ.
6. **Solution:** Evaluate at $(0, 0), (5, 0), (4, 3), (0, 5)$. $Z(0, 0) = 0, Z(5, 0) = 15, Z(4, 3) = 12 + 6 = 18, Z(0, 5) = 10$. Max at $(4, 3)$.
7. **Solution:** $x - y \le -1 \Rightarrow y - x \ge 1$. $-x + y \le 0 \Rightarrow y \le x$. y cannot be both $\ge x + 1$ and $\le x$. Max value does not exist (Infeasible).
8. **Solution:** The region is a pentagon with vertices $(0, 0), (3, 0), (3, 2), (2, 3), (0, 3)$. Area = Area of square (3×3) - Area of triangle at corner $= 9 - 0.5(1)(1) = 8.5$.
9. **Solution:** $x+y \le 12$ intersects axes at $(12, 12)$. $2x+y \le 20$ intersects at $(10, 20)$. $x+y \le 12$ is the redundant constraint because the region is primarily bounded by $2x + y \le 20$ and axes near the origin.
10. **Solution:** Slope of Z is $-1/k$. Slope of segment $(4, 0) - (4, 3)$ is ∞. Slope of $(4, 3) - (0, 5)$ is -0.5. For $(4, 3)$ to be the only max, $-0.5 < -1/k < \infty$.
11. **Solution:** Corners: $(0, 8), (5, 3), (10, 0)$. $Z(0, 8) = 16, Z(5, 3) = 15 + 6 = 21, Z(10, 0) = 30$. Min = 16.
12. **Solution:** $Z(0, 0) = 0, Z(6, 0) = 12, Z(0, 4) = 12$. Max value = 12.
13. **Solution:** Check $(2, 2)$ in $x + y \ge 5$: $2 + 2 = 4 < 5$. The point lies outside (does not satisfy constraints).
14. **Solution:** Corners: $(0, 0), (2, 0), (5/19, 54/19)$ approx? No, intersection $(20/19, 45/19)$. $Z(2, 0) = 10, Z(0, 3) = 9, Z(20/19, 45/19) = (100 + 135)/19 = 12.36$. Max = 12.36.
15. **Solution:** Since the objective function $Z = x + y$ is parallel to the constraint $x + y = 10$, there are infinite optimal solutions on the line segment between $(10, 0)$ and $(0, 10)$.
16. **Solution:** $x + y \ge 3 + 2 = 5$. Combined with $x + y \le 5$, the feasible region is the single point $(3, 2)$.
17. **Solution:** Corners: $(0, 0), (30, 0), (20, 30), (0, 50)$. $Z(30, 0) = 120, Z(20, 30) = 80 + 30 = 110, Z(0, 50) = 50$. Max = 120.
18. **Solution:** $Z = 2x - y$. Corners: $(0, 0), (5, 0), (2, 3), (0, 4)$. $Z(0, 0) = 0, Z(5, 0) = 10, Z(2, 3) = 1, Z(0, 4) = -4$. Min at $(0, 4)$.
19. **Solution:** Corners: $(0, 0), (10, 0), (0, 10)$. Z values: $0, 30, 50$. Average $= (0 + 30 + 50)/3 = 80/3 = 26.67$.
20. **Solution:** For all points to be optimal, the slope of Z must equal the slope of the line $x + y = 10$. Slope of $x + y = 10$ is -1. Slope of Z is $-a/b$. $-1/b = -1 \Rightarrow b = 1$.