Chapter: Integration and its Applications

SOLUTIONS

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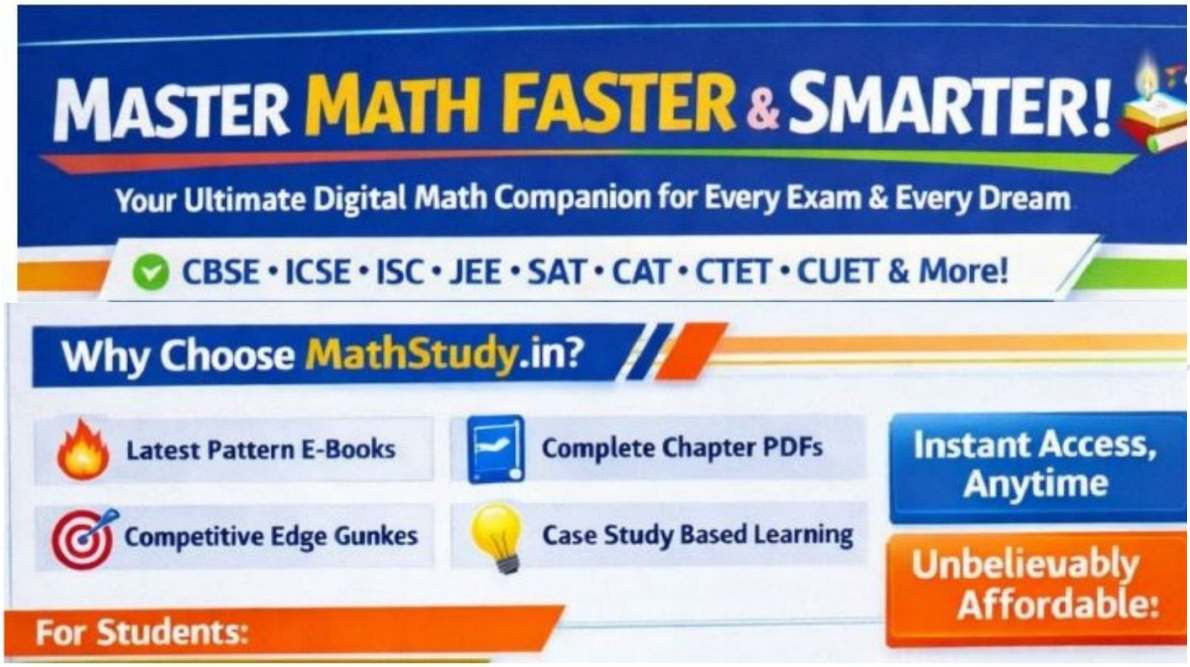

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Solutions

from 0 to 1.5]

1. **Solution:** Multiply and divide by $\cos x$. Let $u = x\sec x$ and $v = \frac{\cos x}{(x\sin x+\cos x)^2}$. Use Integration by Parts. **Ans:** $\frac{\sin x - x\cos x}{x\sin x+\cos x} + C$.

2. **Solution:** Divide by $\cos^2 x$ in denominator and numerator. Let $\tan x = t^2 \Rightarrow \sec^2 x dx = 2tdt$. Integral becomes $\int \frac{t}{t^2} \cdot 2tdt = 2\int dt$. **Ans:** $2\sqrt{\tan x} + C$.

3. **Solution:** Take x^4 out of the bracket: $\int \frac{dx}{x^5(1+x^{-4})^{3/4}}$. Let $1 + x^{-4} = t^4$. **Ans:** $-(1 + x^{-4})^{1/4} + C$.

4. **Solution:** Write $\frac{x^2+1}{(x+1)^2}$ as $\frac{x^2-1+2}{(x+1)^2} = \frac{x-1}{x+1} + \frac{2}{(x+1)^2}$. This is $f(x) + f'(x)$ form. **Ans:** $e^x \frac{x-1}{x+1} + C$.

5. **Solution:** Use $2\cos A\cos B$ formulas and simplify the trigonometric ratio. **Ans:** $-(\sin 2x + \sin x) + C$.

6. **Solution:** Use $x \to \pi - x$ property. $2I = \pi\int_0^\pi \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$. Change limits to $2 \times \pi/2$ and divide by $\cos^2 x$. **Ans:** $\frac{\pi^2}{2ab}$.

7. **Solution:** Use $x \to \pi/2 - x$. $2I = \int \frac{\sin^2 x+\cos^2 x}{\sin x+\cos x}dx$. Multiply and divide by $\sqrt{2}$. **Ans:** $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$.

8. **Solution:** Let $I = \int_0^{\pi/4} \log(1 + \tan(\frac{\pi}{4} - x))dx = \int \log(\frac{2}{1+\tan x})dx = \int \log 2dx - I$. **Ans:** $\frac{\pi}{8}\log 2$.

9. **Solution:** Write $\cot^{-1}$ as $\tan^{-1}(\frac{1}{1-x+x^2}) = \tan^{-1}(\frac{x-(x-1)}{1+x(x-1)}) = \tan^{-1} x - \tan^{-1}(x-1)$. **Ans:** $\frac{\pi}{2} - \log 2$.

10. **Solution:** Use $x = 1/t$ to show the integral is independent of n when combined. **Ans:** $\pi/4$.

11. **Solution:** Points: $(0,0), (4,4)$. Area $= \int_0^4 (2\sqrt{x} - x)dx = [\frac{4}{3}x^{3/2} - \frac{x^2}{2}]_0^4 = 32/3 - 8$. **Ans:** $8/3$ sq. units.

12. **Solution:** Intersection at $x = 1$. Area $= 2[\int_0^1 \sqrt{4-(x-2)^2}dx + \int_1^2 \sqrt{4-x^2}dx]$. **Ans:** $(\frac{8\pi}{3} - 2\sqrt{3})$ sq. units.

13. **Solution:** Area $= \int_0^3 [\frac{2}{3}\sqrt{9-x^2} - 2(1 - \frac{x}{3})]dx$. **Ans:** $\frac{3}{2}(\pi - 2)$ sq. units.

14. **Solution:** Points of intersection $(-1, 2)$ and $(2, 1)$. Area is found by splitting at $x = 0, 1$. **Ans:** 4 sq. units.

15. **Solution:** Write denominator as $(\cos^2 x + \sin^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$. Divide by $\cos^6 x$. **Ans:** $\tan^{-1}(\tan x - \cot x) + C$.

16. **Solution:** Area $= \int_1^e \log x dx = [x\log x - x]_1^e = (e - e) - (0 - 1)$. **Ans:** 1 sq. unit.

17. **Solution:** $2x + 1 = A(2x + 4) + B \Rightarrow A = 1, B = -3$. Split into two integrals. **Ans:** $2\sqrt{x^2+4x+3} - 3\log|x + 2 + \sqrt{x^2+4x+3}| + C$.

18. **Solution:** Split limits: $\int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{1.5} 2dx$. **Ans:** $3 - \sqrt{2}$.

19. **Solution:** Find equations of sides AB, BC, CA and integrate $\int (y_{upper} - y_{lower})dx$. **Ans:** 4 sq. units.

20. **Solution:** Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)dx = dt$. Also $t^2 = 1 + \sin 2x$. **Ans:** $-\log|\sin x + \cos x + \sqrt{\sin 2x}| + C$.