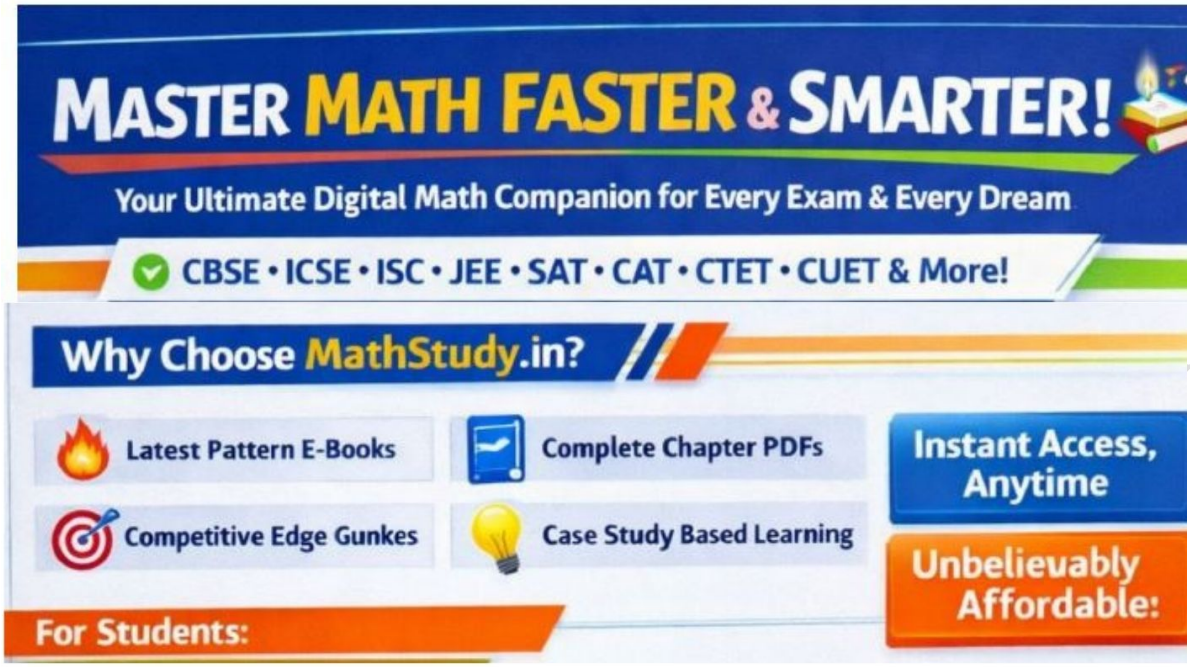
Chapter: Differential Equations (Order, Degree, and Variable Separable)



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Solutions

1. **Solution:** $y(y') = x(y')^2 + 2$. Highest derivative is y' (Order 1). Its highest power is 2. **Ans:** Order = 1, Degree = 2.

2. **Solution:** Cube both sides: $(y'')^3 = 1 + (y')^2$. **Ans:** Order = 2, Degree = 3.

3. **Solution:** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Differentiating twice to eliminate two constants. **Ans:** $xy\frac{d^2y}{dx^2} + x(\frac{dy}{dx})^2 - y\frac{dy}{dx} = 0$.

4. **Solution:** $y = c_1e^{c_2}e^x + c_3e^{-x}$. Since $c_1e^{c_2}$ is just one constant A, $y = Ae^x + c_3e^{-x}$. **Ans:** Order = 2, Degree = 1.

5. **Solution:** $e^y dy = (e^{2x} - e^xe^y)dx$. This is not simple variable separable. Rearrange: $\frac{dy}{dx} = e^{2x-y} - e^x$. **Ans:** $e^y = e^x + Ce^{-e^x}$. (Wait: In variable separable context, use $y' = e^x/e^y(e^x - e^y)$).

6. **Solution:** $\frac{dy}{y-1} = \frac{dx}{x(x+1)} = (\frac{1}{x} - \frac{1}{x+1})dx$. Integrating: $\log|y-1| = \log|\frac{x}{x+1}| + \log C$. At $(1,0)$, $C = -2$. **Ans:** $(y-1)(x+1) = -2x$.

7. **Solution:** $\frac{dy}{dx} = \frac{x(x+y+1)}{x(x+1)} = \frac{x+y+1}{x+1} = 1+\frac{y}{x+1}$. Let $x+1 = X$. **Ans:** $y = (x+1)\log|C(x+1)|$.

8. **Solution:** Let $x + y = v \Rightarrow 1 + y' = v'$. $v' - 1 = 1/\cos v \Rightarrow \frac{\cos v}{1+\cos v}dv = dx$. **Ans:** $(x+y) - 2\tan(\frac{x+y}{2}) = x + C \Rightarrow y - 2\tan(\frac{x+y}{2}) = C$.

9. **Solution:** $\frac{dy}{1+y^2} = -\frac{e^x}{1+e^{2x}}dx$. Integrating: $\tan^{-1}y = -\tan^{-1}e^x + C$. At $x = 0, y = 1$: $\pi/4 = -\pi/4 + C \Rightarrow C = \pi/2$. **Ans:** $\tan^{-1}y + \tan^{-1}e^x = \pi/2$ or $ye^x = 1$.

10. **Solution:** $\frac{dy}{dx} = \sin^{-1}x$. Highest derivative is y'. **Ans:** Order = 1, Degree = 1.

11. **Solution:** $(y - ay^2)dx = (x + a)dy \Rightarrow \frac{dx}{x+a} = \frac{dy}{y(1-ay)}$. Use partial fractions. **Ans:** $y = \frac{x+a}{c+a(x+a)}$.

12. **Solution:** $\frac{dy}{\sin y} = \frac{dx}{\cos x+\sin x}$. $\int \csc y dy = \frac{1}{\sqrt{2}}\int \sec(x - \pi/4)dx$. **Ans:** $\log|\tan(y/2)| = \frac{1}{\sqrt{2}}\log|\tan(x/2 + \pi/8)| + C$.

13. **Solution:** $\frac{y}{\sqrt{1-y^2}}dy = dx$. Let $1 - y^2 = t$. **Ans:** $-\sqrt{1-y^2} = x + C$.

14. **Solution:** $(x-r)^2 + y^2 = r^2 \Rightarrow x^2 - 2xr + y^2 = 0 \Rightarrow 2r = \frac{x^2+y^2}{x}$. Differentiate to eliminate r. **Ans:** $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$.

15. **Solution:** $\frac{dx}{dy} = x + y + 1 \Rightarrow \frac{dx}{dy} - x = y + 1$. (Linear in x). **Ans:** $x = -y - 2 + Ce^y$.

16. **Solution:** $\frac{d}{dx}[y(1+x^2)] = \frac{1}{1+x^2}$. Integrating: $y(1+x^2) = \tan^{-1}x + C$. At $(1,0)$, $C = -\pi/4$. **Ans:** $y(1 + x^2) = \tan^{-1}x - \pi/4$.

17. **Solution:** $x - y = v \Rightarrow 1 - y' = v' \Rightarrow 1 - v' = \sec^2 v \Rightarrow v' = 1 - \sec^2 v = -\tan^2 v$. **Ans:** $\cot(x - y) = x + C$.

18. **Solution:** Tangent equation: $Y - y = m(X - x)$. Intercepts: $x - y/m$ and $y - mx$. Sum $= x - y/m + y - mx = k$. **Ans:** $\sqrt{x} + \sqrt{y} = \sqrt{k}$.

19. **Solution:** $v + x\frac{dv}{dx} = \frac{v^2x^2+2vx^2}{x^2} = v^2 + 2v \Rightarrow x\frac{dv}{dx} = v^2 + v \Rightarrow \frac{dv}{v(v+1)} = \frac{dx}{x}$. **Ans:** $\frac{y}{x+y} = Cx$.

20. **Solution:** Squaring: $[1 + (y')^2]^3 = k^2(y'')^2$. **Ans:** Order = 2, Degree = 2.