

## Unit II: Algebra - Determinants

### General Instructions

1. Total Questions: **20**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Read each question carefully before answering.
5. Choose the most appropriate answer from the given options.
6. Use of calculator or electronic devices is strictly prohibited.

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- Let  $A$  be a  $3 \times 3$  matrix such that  $|A| = 2$ . If  $B = 2A^{-1}$ , find the value of  $|\text{adj}(B)|$ .
- Find the value of  $\theta \in (0, \pi)$  for which the matrix  $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$  has a determinant equal to 3.
- If  $f(x) = \begin{vmatrix} x-3 & 2x^2-1 & x^3+1 \\ 2x-1 & x-3 & x^2+2 \\ 3x^2+2 & 2x & x \end{vmatrix}$ , find the value of  $f'(0)$ .
- Show that the system of equations  $x + y + z = 2$ ,  $x + 2y + 3z = 5$ ,  $x + 3y + kz = 8$  has a unique solution for all  $k \neq 5$ . What happens when  $k = 5$ ?
- If  $A$  is a square matrix of order 3 and  $A^2 = 4I$ , find the possible values of  $|\text{adj}(A)|$ .
- Using properties of determinants, prove that  $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ .
- If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$ , find  $A^{-1}$  and use it to solve the system:  $x - y + z = 4$ ,  $2x + 3y = 1$ ,  $18x + 2y + 10z = 2$ .
- Let  $x, y, z$  be real numbers such that  $x + y + z = \pi$ . Find the value of the determinant:  $\begin{vmatrix} \sin(x+y+z) & \sin y & \cos z \\ -\sin y & 0 & \tan x \\ \cos(x+y) & \tan x & 0 \end{vmatrix}$ .
- If  $A$  is a non-singular matrix of order  $n$ , prove that  $|\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$ .
- Find the value of  $\lambda$  such that the system  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = 12$  is inconsistent.
- If  $a, b, c$  are in A.P., find the value of  $\begin{vmatrix} x+2 & x+3 & x+a \\ x+3 & x+4 & x+b \\ x+4 & x+5 & x+c \end{vmatrix}$ .
- If  $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ , verify that  $A \cdot (\text{adj} A) = |A|I$ .
- Find the area of the triangle whose vertices are the roots of the equation  $z^3 + iz^2 + 2iz = 0$  in the complex plane using determinants.
- Let  $A$  be a  $3 \times 3$  matrix such that  $A^T A = I$ . Show that  $|A| = \pm 1$  and find  $|\text{adj}(A^T)|$ .
- If  $\Delta(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ , show that  $\Delta(x)$  is independent of  $x$ .
- Find the inverse of the matrix  $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ .

17. If  $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ , find a matrix  $B$  such that  $AB = I$  and show that  $B = \frac{1}{2}adj(A)$ .

18. For a  $3 \times 3$  matrix  $A$ , if  $|A| = 4$ , find the value of  $|det(2 \cdot adjA)|$ .

19. If  $x + y + z = 0$ , solve for  $x$  in the equation:  $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = ka^2b^2c^2$ . Find the value of  $k$ .

20. Show that if  $A$  is an orthogonal matrix, then  $A^{-1}$  is also orthogonal.

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