Unit II: Algebra - Determinants

SOLUTIONS

www.udgamwelfarefoundation.com

For Best Mathematics E-Books, Visit:
www.mathstudy.in

Solutions

1. **Solution:** $|B| = |2A^{-1}| = 2^3 \cdot \frac{1}{|A|} = \frac{8}{2} = 4$. Then $|adj(B)| = |B|^{3-1} = 4^2 = 16$.

2. **Solution:** Expanding $|A| = 1(1+\sin^2\theta) - \sin\theta(-\sin\theta+\sin\theta) + 1(\sin^2\theta+1) = 2(1+\sin^2\theta)$. Set $2(1+\sin^2\theta) = 3 \Rightarrow \sin^2\theta = 1/2 \Rightarrow \sin\theta = 1/\sqrt{2}$. Thus $\theta = \pi/4$ or $3\pi/4$.

3. **Solution:** $f'(0)$ is found by differentiating one row at a time and evaluating at $x = 0$. Sum of the three resulting determinants gives $f'(0)$.

4. **Solution:** $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & k \end{vmatrix} = 1(2k-9) - 1(k-3) + 1(3-2) = k-5$. Unique solution if $k - 5 \neq 0$. If $k = 5$, $\Delta = 0$, then test $\Delta_x, \Delta_y, \Delta_z$ for consistency (infinitely many or no solution).

5. **Solution:** $A^2 = 4I \Rightarrow |A|^2 = |4I| = 4^3 = 64 \Rightarrow |A| = \pm 8$. $|adj(A)| = |A|^{3-1} = (\pm 8)^2 = 64$.

6. **Solution:** Factor out a, b, c from C_1, C_2, C_3 respectively. Perform $R_1 \to R_1 - (R_2 + R_3)$ to simplify.

7. **Solution:** Calculate $|A|$ and $adj(A)$. $X = A^{-1}B$.

8. **Solution:** $\sin(x+y+z) = \sin(\pi) = 0$. The determinant becomes skew-symmetric with zeros on diagonal, but check carefully. Since $\sin(\pi) = 0$ and $\cos(x+y) = \cos(\pi - z) = -\cos z$, the determinant is zero.

9. **Solution:** Property derivation: $A \cdot adj A = |A|I \Rightarrow |adj A| = |A|^{n-1}$. Replacing A with $adj A$ gives the result.

10. **Solution:** $\Delta = 0$ for inconsistency or infinite solutions. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$. If $\lambda = 3$, $\Delta_z \neq 0$, so system is inconsistent.

11. **Solution:** $2b = a+c$. Perform $R_1 + R_3 - 2R_2$. The row becomes $[0, 0, x+a+x+c-2(x+b)] = [0, 0, a+c-2b] = [0, 0, 0]$. Value is 0.

12. **Solution:** $|A| = 1(15-1) - 2(10-1) + 1(2-3) = 14 - 18 - 1 = -5$. Calculate cofactors to form $adj A$.

13. **Solution:** $z(z^2 + iz + 2i) = 0 \Rightarrow z_1 = 0, z_2, z_3$ from quadratic. Use Area $= \frac{1}{2}|\text{Im}(\bar{z_1}z_2 + \dots)|$ or standard determinant with (x, y) coordinates.

14. **Solution:** $|A^T A| = |I| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$. $|adj(A^T)| = |A^T|^2 = (\pm 1)^2 = 1$.

15. **Solution:** Use addition formulas for cos and sin. Differentiate w.r.t x to show $\Delta'(x) = 0$.

16. **Solution:** $|A| = 1$. $A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

17. **Solution:** $B = A^{-1}$. $|A| = 14 - 12 = 2$. $adj A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$. $B = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$.

18. **Solution:** $|2 \cdot adj A| = 2^3 |adj A| = 8 \cdot |A|^2 = 8 \cdot 16 = 128$.

19. **Solution:** By expanding and comparing coefficients, $k = 2$.

20. **Solution:** $A^T A = I \Rightarrow A^{-1} = A^T$. Then $(A^{-1})^T(A^{-1}) = (A^T)^T A^T = AA^T = I$.