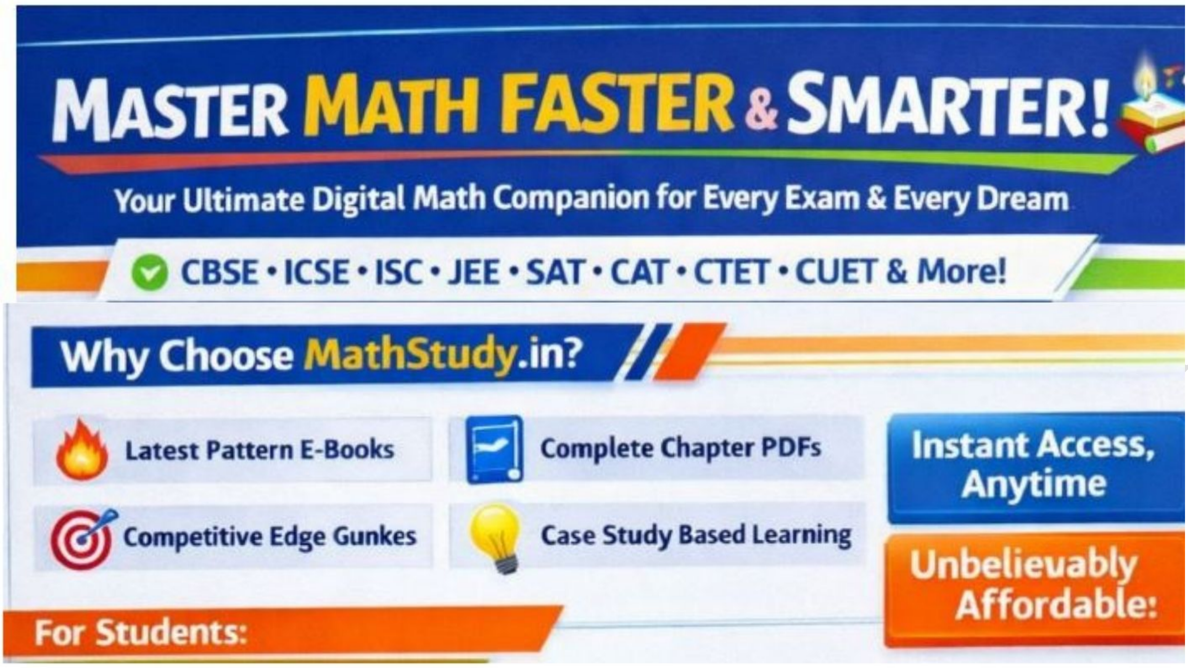
Chapter: Calculus (Higher Order Derivatives, Monotonicity, Maxima/Minima)

SOLUTIONS



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Solutions

1. $f(x)$ is a polynomial, continuous on $[1, 3]$ and differentiable on $(1, 3)$. $f(1) = f(2) = 0$ and $f(2) = f(3) = 0$. By Rolle's Theorem, there is $c_1 \in (1, 2)$ s.t. $f'(c_1) = 0$ and $c_2 \in (2, 3)$ s.t. $f'(c_2) = 0$.

2. $y_1 = m(x+\sqrt{x^2+1})^{m-1}(1+\frac{x}{\sqrt{x^2+1}}) = \frac{my}{\sqrt{x^2+1}}$. Squaring: $(x^2+1)y_1^2 = m^2y^2$. Differentiating again: $(x^2+1)2y_1y_2 + 2xy_1^2 = 2m^2yy_1$. Divide by $2y_1$ to get the result.

3. $f'(x) = 4 - \frac{1}{x^2}$. For strictly decreasing, $4 - \frac{1}{x^2} < 0 \Rightarrow 4 < \frac{1}{x^2} \Rightarrow x^2 < 1/4 \Rightarrow x \in (-1/2, 0) \cup (0, 1/2)$.

4. $f'(x) = 3(a+2)x^2 - 6ax + 9a$. For strictly decreasing, $f'(x) < 0$ for all x. This requires $a + 2 < 0$ and Discriminant $D < 0$. $36a^2 - 4(3(a+2))(9a) < 0 \Rightarrow a^2 - a(a+2) < 0 \Rightarrow -2a < 0 \Rightarrow a > 0$. But $a < -2$. No such a exists.

5. $f'(x)$ has roots at 1, 2, 3 and is of degree 3. $f'(x) = k(x-1)(x-2)(x-3)$. Integrating: $f(x) = k(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x) + C$. $f(0) = 0 \Rightarrow C = 0$.

6. Slope $m = y' = -3x^2 + 6x + 9$. To maximize m, $m' = -6x + 6 = 0 \Rightarrow x = 1$. Max slope $m(1) = -3 + 6 + 9 = 12$.

7. $f'(x) = 3x^2 + 2px + q$. For always increasing, $f'(x) \geq 0 \forall x$. This requires $D \leq 0 \Rightarrow (2p)^2 - 4(3)(q) \leq 0 \Rightarrow p^2 \leq 3q$.

8. Let x be length for square ($side = x/4$) and $28 - x$ for circle ($radius = \frac{28-x}{2\pi}$). Area $A = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$. $dA/dx = 0 \Rightarrow x = \frac{112}{4+\pi}$.

9. Minimize $D^2 = (x-2)^2 + (y-1)^2$ where $x = y^2/4$. $g(y) = (\frac{y^2}{4} - 2)^2 + (y-1)^2$. $g'(y) = 0$ gives $y = 2$. Point is $(1, 2)$.

10. $f'(x) = \frac{e^x(\cos x - \sin x) - e^x(\sin x + \cos x)}{e^{2x}} = \frac{-2\sin x}{e^x}$. $f''(x) = \frac{-2e^x \cos x + 2e^x \sin x}{e^{2x}}$. $f''(\pi/2) = \frac{2}{e^{\pi/2}}$.

11. $V = \frac{1}{3}\pi(l \sin\alpha)^2(l\cos\alpha) = \frac{1}{3}\pi l^3 \sin^2\alpha\cos\alpha$. $dV/d\alpha = 0 \Rightarrow \sin\alpha(2\cos^2\alpha - \sin^2\alpha) = 0 \Rightarrow \tan^2\alpha = 2 \Rightarrow \alpha = \tan^{-1}\sqrt{2}$.

12. $f'(x) = -2xe^{-x^2}$. $f''(x) = (4x^2 - 2)e^{-x^2}$. $f''(x) = 0 \Rightarrow 4x^2 = 2 \Rightarrow x = \pm 1/\sqrt{2}$.

13. $f(x) = x^3$ for $x \geq 0$ and $-x^3$ for $x < 0$. $f'(x) = 3x^2$ for $x \geq 0$ and $-3x^2$ for $x < 0$. $f''(x) = 6x$ for $x \geq 0$ and $-6x$ for $x < 0$. $f''(x) = 6|x|$.

14. $f'(x) = 1 - \frac{1}{1+x} = \frac{x}{1+x}$. Increasing when $x/(1+x) > 0$. For $x > -1$, this means $x > 0$. **Answer:** $(0, \infty)$.

15. $f'(x) = 6x^2 - 18ax + 12a^2 = 6(x-a)(x-2a)$. Roots $a, 2a$. Since $a > 0$, max at $p = a$, min at $q = 2a$. $a^2 = 2a \Rightarrow a = 2$ (since $a > 0$).

16. Let sides be $2r\cos\theta, 2r\sin\theta$. Area $A = 4r^2\sin\theta\cos\theta = 2r^2\sin 2\theta$. Max area is $2r^2$ when $\theta = \pi/4$ (a square).

17. Let $g(x) = x - \log(1+x)$. $g'(x) = x/(1+x) > 0$ for $x > 0 \Rightarrow \log(1+x) < x$. Let $h(x) = \log(1+x) - (x - x^2/2)$. $h'(x) = \frac{x^2}{1+x} > 0$ for $x > 0 \Rightarrow \log(1+x) > x - x^2/2$.

18. Standard differentiation using chain rule and quotient rule; rearrange to form the given differential equation.

19. $f'(x) = 2\cos 2x - 1 = 0 \Rightarrow x = \pi/6, 5\pi/6$. Max value at $x = \pi/6$ is $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$. Min value at $x = 5\pi/6$ is $-\frac{\sqrt{3}}{2} - \frac{5\pi}{6}$.

20. Perimeter $2r + 2h + \pi r = 10$. Area $A = 2rh + \frac{1}{2}\pi r^2$. Substituting h and differentiating gives $r = \frac{10}{\pi+4}$. **Answer: Radius $r = \frac{10}{\pi+4}$ m, Height $h = \frac{10}{\pi+4}$ m.**