

Unit III: Calculus - Continuity and Differentiability

SOLUTIONS

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Solutions

- Solution:** LHL: $\lim_{x \rightarrow 0^-} \frac{2 \sin^2 2x}{x^2} = 2 \cdot 4 = 8$. RHL: $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}(\sqrt{16+\sqrt{x}+4})}{(\sqrt{16+\sqrt{x}-4})(\sqrt{16+\sqrt{x}+4})} = \lim_{x \rightarrow 0^+} \frac{\sqrt{x}(\sqrt{16+\sqrt{x}+4})}{\sqrt{x}} = 8$. For continuity, $a = 8$.
- Solution:** For continuity at $x = 1$: $1 + 3 + a = b + 2 \Rightarrow a - b = -2$. For differentiability: $LHD = 2x + 3|_{x=1} = 5$; $RHD = b$. Thus $b = 5$ and $a = 5 - 2 = 3$.
- Solution:** $y^2 = \log x + y$. Diff. w.r.t x : $2yy' = \frac{1}{x} + y' \Rightarrow y'(2y - 1) = \frac{1}{x}$.
- Solution:** $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(h) + xh^2 + x^2h}{h} = \lim_{h \rightarrow 0} \left(\frac{f(h)}{h} + xh + x^2 \right) = 1 + x^2$.
- Solution:** $dx/dt = a(-\sin t + \sin t + t \cos t) = at \cos t$. $dy/dt = a(\cos t - \cos t + t \sin t) = at \sin t$. $dy/dx = \tan t$. $d^2y/dx^2 = \sec^2 t \cdot \frac{dt}{dx} = \sec^2 t \cdot \frac{1}{at \cos t} = \frac{\sec^3 t}{at}$.
- Solution:** Let $x = \tan \theta$. $u = \tan^{-1}(\frac{\sec \theta - 1}{\tan \theta}) = \theta/2$. $v = \sin^{-1}(\frac{2 \tan \theta}{1 + \tan^2 \theta}) = 2\theta$. $du/dv = \frac{1/2}{2} = 1/4$.
- Solution:** Take log: $x \log y = y - x$. Diff. w.r.t x : $\log y + \frac{x}{y} y' = y' - 1 \Rightarrow y'(1 - \frac{x}{y}) = 1 + \log y$. Since $x = \frac{y}{1 + \log y}$, substituting gives the result.
- Solution:** For $x \in (-1, 1)$, $f(x) = (1 - x) + (x + 1) = 2$. $f'(0) = 0$. At $x = 1$, LHD is 0, RHD is 2 ($f(x) = 2x$ for $x > 1$). Not differentiable at $x = 1$.
- Solution:** $y_1 = m(x + \sqrt{x^2 + 1})^{m-1} (1 + \frac{x}{\sqrt{x^2 + 1}}) = \frac{my}{\sqrt{x^2 + 1}}$. Squaring: $(x^2 + 1)y_1^2 = m^2 y^2$. Differentiating again yields the differential equation.
- Solution:** Potential discontinuities for $[x^2]$ in $(1, 2)$ are at $x^2 = 2, 3 \Rightarrow x = \sqrt{2}, \sqrt{3}$. However, at these points $\sin(\pi x) \neq 0$, so jump discontinuities remain.
- Solution:** $dx/d\theta = \sec \theta \tan \theta + \sin \theta$. $dy/d\theta = n \sec^n \theta \tan \theta + n \cos^{n-1} \theta \sin \theta$. Using $dy/dx = (dy/d\theta)/(dx/d\theta)$ and squaring leads to the result.
- Solution:** $f'(x) = \frac{1}{x \log x}$. $f''(x) = -\frac{(1 + \log x)}{(x \log x)^2}$. At $x = e$, $f''(e) = -\frac{2}{e^2}$.
- Solution:** $f'(x) = f(x)f'(0)$. $f'(5) = f(5)f'(0) = 2 \times 3 = 6$.
- Solution:** Let $x = \sin \theta$ and $2/\sqrt{13} = \sin \alpha$, $3/\sqrt{13} = \cos \alpha$. $y = \cos^{-1}(\sin(\theta - \alpha)) = \cos^{-1}(\cos(\pi/2 - \theta + \alpha)) = \pi/2 - \sin^{-1} x + \alpha$. $dy/dx = -1/\sqrt{1 - x^2}$.
- Solution:** Continuity: $\lim_{x \rightarrow 0} x \sin(1/x) = 0$. Differentiability: $\lim_{h \rightarrow 0} \frac{h \sin(1/h) - 0}{h} = \lim_{h \rightarrow 0} \sin(1/h)$, which does not exist (oscillates).
- Solution:** $y_1 = \frac{1}{1+x^2} \Rightarrow (1+x^2)y_1 = 1$. Diff: $(1+x^2)y_2 + 2xy_1 = 0$. Diff again: $(1+x^2)y_3 + 2xy_2 + 2y_1 = 0 \Rightarrow (1+x^2)y_3 + 4xy_2 + 2y_1 = 0$.
- Solution:** $\lim_{x \rightarrow \pi/2} \frac{k \cos x}{\pi - 2x}$. Using L'Hôpital: $\lim_{x \rightarrow \pi/2} \frac{-k \sin x}{-2} = k/2$. Set $k/2 = 3 \Rightarrow k = 6$.
- Solution:** $e^x + e^y = e^x \cdot e^y \Rightarrow e^{-y} + e^{-x} = 1$. Diff: $-e^{-y} y' - e^{-x} = 0 \Rightarrow y' = -e^{-x}/e^{-y} = -e^{y-x}$.
- Solution:** $x = x^3 \Rightarrow x = 0, 1, -1$. Check LHD/RHD at these intersection points. Sharp corners occur at $x = 1$ and $x = -1$.
- Solution:** $y_1 = \frac{\cos(\log x)}{x} \Rightarrow xy_1 = \cos(\log x)$. Diff: $xy_2 + y_1 = -\frac{\sin(\log x)}{x}$. $x^2 y_2 + xy_1 = -y \Rightarrow x^2 y_2 + xy_1 + y = 0$.