Unit III: Calculus - Indefinite Integrals

General Instructions

1. Total Questions: **20**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Read each question carefully before answering.
5. Choose the most appropriate answer from the given options.
6. Use of calculator or electronic devices is strictly prohibited.

1. Evaluate: $\int \frac{x^2-1}{x^4+x^2+1}dx$
2. Find the integral of $\frac{1}{\cos^6 x+\sin^6 x}dx$.
3. Evaluate: $\int \frac{dx}{(x+1)\sqrt{x^2-1}}$
4. Find $\int \sqrt{\tan x}dx$.
5. Evaluate: $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$
6. Find the value of $\int \frac{dx}{x(x^n+1)}$ for any positive integer n.
7. Evaluate: $\int \frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx$
8. Find $\int \frac{dx}{(x^2+1)^2}$ using trigonometric substitution.
9. Evaluate: $\int \frac{e^x(1+x)}{(2+x)^2}dx$
10. Find the integral $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx$.
11. Evaluate: $\int \frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$
12. Find $\int \frac{dx}{\sin(x-a)\sin(x-b)}$.
13. Evaluate: $\int \frac{2x+3}{x^2+x+1}dx$
14. Find the integral of $\frac{\log x}{(1+\log x)^2}dx$.
15. Evaluate: $\int \frac{dx}{x^4+1}$
16. Find $\int \frac{\sin x}{\sin 3x}dx$.
17. Evaluate: $\int \frac{dx}{5+4\cos x}$ using the half-angle substitution.
18. Find $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}dx$.
19. Evaluate: $\int \frac{dx}{\sqrt{(x-a)(b-x)}}$ where $b > a$.
20. Find $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$.

Solutions

1. **Solution:** Divide by x^2: $\int \frac{1-1/x^2}{x^2+1+1/x^2}dx = \int \frac{d(x+1/x)}{(x+1/x)^2-1}$. Let $t = x + 1/x$. Result: $\frac{1}{2}\log|\frac{x+1/x-1}{x+1/x+1}| + C$.

2. **Solution:** $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x\cos^2 x = 1 - \frac{3}{4}\sin^2 2x$. Divide numerator and denominator by $\cos^6 x$ or use $\tan x$ substitution. Result: $\tan^{-1}(\frac{\tan^2 x - 1}{\sqrt{3}\tan x}) + C$ (after simplification).

3. **Solution:** Put $x + 1 = 1/t$. $dx = -1/t^2 dt$. Radical becomes $\sqrt{(1/t-1)^2 - 1}$. Result: $-\sqrt{\frac{x-1}{x+1}} + C$.

4. **Solution:** Put $\tan x = t^2$, $dx = \frac{2t}{1+t^4}dt$. Integral becomes $\int \frac{2t^2}{t^4+1}dt$. Split into $\frac{t^2+1}{t^4+1} + \frac{t^2-1}{t^4+1}$ and solve. Result: $\frac{1}{\sqrt{2}}\tan^{-1}(\frac{\tan x - 1}{\sqrt{2\tan x}}) + \frac{1}{2\sqrt{2}}\log|\frac{\tan x - \sqrt{2\tan x}+1}{\tan x + \sqrt{2\tan x}+1}| + C$.

5. **Solution:** Use parts. $u = x/\cos x, dv = \frac{\cos x}{(x\sin x + \cos x)^2}dx$. $v = \frac{-1}{x\sin x + \cos x}$. Result: $\frac{\sin x - x\cos x}{x\sin x + \cos x} + C$.

6. **Solution:** Multiply and divide by x^{n-1}: $\int \frac{x^{n-1}}{x^n(x^n+1)}dx$. Put $x^n = t$. Result: $\frac{1}{n}\log|\frac{x^n}{x^n+1}| + C$.

7. **Solution:** Multiply by -1: $\int \frac{\cos x - \sin x}{\sqrt{1-(\sin x + \cos x)^2}}dx$. Let $t = \sin x + \cos x$. Result: $-\sin^{-1}(\sin x + \cos x) + C$.

8. **Solution:** Put $x = \tan\theta, dx = \sec^2\theta d\theta$. $\int \frac{\sec^2\theta}{\sec^4\theta}d\theta = \int \cos^2\theta d\theta$. Result: $\frac{1}{2}[\tan^{-1}x + \frac{x}{1+x^2}] + C$.

9. **Solution:** $\int e^x[\frac{1}{x+2} - \frac{1}{(x+2)^2}]dx$. Matches $e^x(f(x) + f'(x))$. Result: $\frac{e^x}{x+2} + C$.

10. **Solution:** Divide by x^2 inside and outside: $\int \frac{1-1/x^2}{(x+1/x)\sqrt{x^2+1/x^2}}dx$. Put $t = x + 1/x$. Result: $\frac{1}{\sqrt{2}}\sec^{-1}(\frac{x+1/x}{\sqrt{2}}) + C$.

11. **Solution:** Use $2\cos A\cos B$ identities and simplify numerator to $-(1 - 2\cos 3x)(\cos 2x + \cos x)$. Result: $-(\frac{\sin 2x}{2} + \sin x) + C$.

12. **Solution:** Multiply by $\frac{\sin(a-b)}{\sin(a-b)}$. Use $\sin((x-b)-(x-a))$ in numerator. Result: $\frac{1}{\sin(a-b)}\log|\frac{\sin(x-a)}{\sin(x-b)}| + C$.

13. **Solution:** Write $2x + 3 = (2x + 1) + 2$. Result: $\log|x^2 + x + 1| + \frac{4}{\sqrt{3}}\tan^{-1}(\frac{2x+1}{\sqrt{3}}) + C$.

14. **Solution:** Put $\log x = t \Rightarrow x = e^t$. $\int \frac{te^t}{(1+t)^2}dt = \int e^t[\frac{1}{1+t} - \frac{1}{(1+t)^2}]dt$. Result: $\frac{x}{1+\log x} + C$.

15. **Solution:** Write $1 = \frac{1}{2}[(x^2+1) - (x^2-1)]$. Integral splits into $\int \frac{x^2+1}{x^4+1}$ and $\int \frac{x^2-1}{x^4+1}$. Result: $\frac{1}{2\sqrt{2}}\tan^{-1}(\frac{x^2-1}{\sqrt{2}x}) + \frac{1}{4\sqrt{2}}\log|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}| + C$.

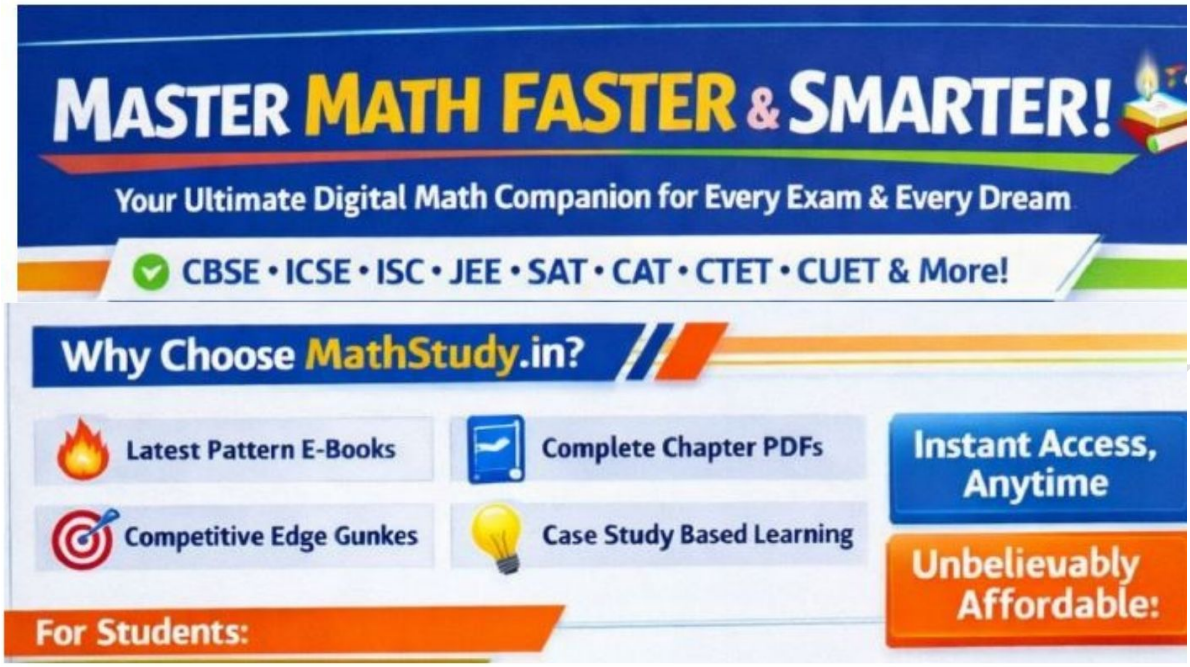
16. **Solution:** $\sin 3x = 3\sin x - 4\sin^3 x$. $\int \frac{1}{3-4\sin^2 x}dx = \int \frac{\sec^2 x}{3\sec^2 x - 4\tan^2 x}dx$. Let $\tan x = t$. Result: $\frac{1}{2\sqrt{3}}\log|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}| + C$.

17. **Solution:** Put $\tan(x/2) = t$. Result: $\frac{2}{3}\tan^{-1}(\frac{1}{3}\tan(x/2)) + C$.

18. **Solution:** $\int \frac{\sqrt{x^2+1}}{x^4}\log(1+1/x^2)dx = \int \frac{1}{x^3}\sqrt{1+1/x^2}\log(1+1/x^2)dx$. Put $1 + 1/x^2 = t^2$. Result: $-\frac{1}{3}(1+1/x^2)^{3/2}[\log(1+1/x^2) - 2/3] + C$.

19. **Solution:** Put $x = a\cos^2\theta + b\sin^2\theta$. Result: $2\sin^{-1}\sqrt{\frac{x-a}{b-a}} + C$.

20. **Solution:** $\int e^{\tan^{-1}x}[1+\frac{x}{1+x^2}]dx$. This is $e^{f(x)}(g'(x)+g(x)f'(x))$ form. Result: $xe^{\tan^{-1}x}+C$.



Special Features

- **Board-Specific** – CBSE, ICSE, ISC, State Boards
- **Exam-Focused** – JEE, SAT, CAT, CTET, CUET, NTSE
- **Grade-Wise** – Class 6 to 12
- **Bilingual Options** – English & Hindi Medium Support
- **Printable & Shareable** – Use offline, anytime

How to Order:

Visit : https://www.mathstudy.in
Browse by Exam, Class, or Topic
Add to Cart & Checkout

Contact & Support:

Email: admin@mathstudy.in
WhatsApp Support Available : +91-+91 92118 65759

Why Wait? Empower your learning journey, save time, and achieve your dreams!

Explore & Start Learning Today:
https://www.mathstudy.in – Premium eBooks for success
https://www.udgamwelfarefoundation.com – Free PDFs, practice tests, & guida

MathStudy.in — Empowering Learners, Enabling Educators, Encouraging Excellence.
Digital Learning | Affordable Excellence | Trusted by Thousands