

Unit III: Calculus - Indefinite Integrals

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Solutions

1. **Solution:** Divide by x^2 : $\int \frac{1-1/x^2}{x^2+1+1/x^2} dx = \int \frac{d(x+1/x)}{(x+1/x)^2-1}$. Let $t = x + 1/x$. Result: $\frac{1}{2} \log \left| \frac{x+1/x-1}{x+1/x+1} \right| + C$.
2. **Solution:** $\cos^6 x + \sin^6 x = 1 - 3 \sin^2 x \cos^2 x = 1 - \frac{3}{4} \sin^2 2x$. Divide numerator and denominator by $\cos^6 x$ or use $\tan x$ substitution. Result: $\tan^{-1} \left(\frac{\tan^2 x - 1}{\sqrt{3} \tan x} \right) + C$ (after simplification).
3. **Solution:** Put $x + 1 = 1/t$. $dx = -1/t^2 dt$. Radical becomes $\sqrt{(1/t - 1)^2 - 1}$. Result: $-\sqrt{\frac{x-1}{x+1}} + C$.
4. **Solution:** Put $\tan x = t^2$, $dx = \frac{2t}{1+t^4} dt$. Integral becomes $\int \frac{2t^2}{t^4+1} dt$. Split into $\frac{t^2+1}{t^4+1} + \frac{t^2-1}{t^4+1}$ and solve. Result: $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2} \tan x} \right) + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + C$.
5. **Solution:** Use parts. $u = x/\cos x$, $dv = \frac{\cos x}{(x \sin x + \cos x)^2} dx$. $v = \frac{-1}{x \sin x + \cos x}$. Result: $\frac{\sin x - x \cos x}{x \sin x + \cos x} + C$.
6. **Solution:** Multiply and divide by x^{n-1} : $\int \frac{x^{n-1}}{x^n(x^n+1)} dx$. Put $x^n = t$. Result: $\frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C$.
7. **Solution:** Multiply by -1 : $\int \frac{\cos x - \sin x}{\sqrt{1 - (\sin x + \cos x)^2}} dx$. Let $t = \sin x + \cos x$. Result: $-\sin^{-1}(\sin x + \cos x) + C$.
8. **Solution:** Put $x = \tan \theta$, $dx = \sec^2 \theta d\theta$. $\int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \int \cos^2 \theta d\theta$. Result: $\frac{1}{2} [\tan^{-1} x + \frac{x}{1+x^2}] + C$.
9. **Solution:** $\int e^x [\frac{1}{x+2} - \frac{1}{(x+2)^2}] dx$. Matches $e^x(f(x) + f'(x))$. Result: $\frac{e^x}{x+2} + C$.
10. **Solution:** Divide by x^2 inside and outside: $\int \frac{1-1/x^2}{(x+1/x)\sqrt{x^2+1/x^2}} dx$. Put $t = x + 1/x$. Result: $\frac{1}{\sqrt{2}} \sec^{-1} \left(\frac{x+1/x}{\sqrt{2}} \right) + C$.
11. **Solution:** Use $2 \cos A \cos B$ identities and simplify numerator to $-(1 - 2 \cos 3x)(\cos 2x + \cos x)$. Result: $-\left(\frac{\sin 2x}{2} + \sin x\right) + C$.
12. **Solution:** Multiply by $\frac{\sin(a-b)}{\sin(a-b)}$. Use $\sin((x-b) - (x-a))$ in numerator. Result: $\frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + C$.
13. **Solution:** Write $2x + 3 = (2x + 1) + 2$. Result: $\log |x^2 + x + 1| + \frac{4}{\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + C$.
14. **Solution:** Put $\log x = t \Rightarrow x = e^t$. $\int \frac{te^t}{(1+t)^2} dt = \int e^t \left[\frac{1}{1+t} - \frac{1}{(1+t)^2} \right] dt$. Result: $\frac{x}{1+\log x} + C$.
15. **Solution:** Write $1 = \frac{1}{2} [(x^2 + 1) - (x^2 - 1)]$. Integral splits into $\int \frac{x^2+1}{x^4+1}$ and $\int \frac{x^2-1}{x^4+1}$. Result: $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2}x} \right) + \frac{1}{4\sqrt{2}} \log \left| \frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1} \right| + C$.
16. **Solution:** $\sin 3x = 3 \sin x - 4 \sin^3 x$. $\int \frac{1}{3-4 \sin^2 x} dx = \int \frac{\sec^2 x}{3 \sec^2 x - 4 \tan^2 x} dx$. Let $\tan x = t$. Result: $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$.
17. **Solution:** Put $\tan(x/2) = t$. Result: $\frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan(x/2) \right) + C$.
18. **Solution:** $\int \frac{\sqrt{x^2+1}}{x^4} \log(1 + 1/x^2) dx = \int \frac{1}{x^3} \sqrt{1 + 1/x^2} \log(1 + 1/x^2) dx$. Put $1 + 1/x^2 = t^2$. Result: $-\frac{1}{3} (1 + 1/x^2)^{3/2} [\log(1 + 1/x^2) - 2/3] + C$.

19. **Solution:** Put $x = a \cos^2 \theta + b \sin^2 \theta$. Result: $2 \sin^{-1} \sqrt{\frac{x-a}{b-a}} + C$.

20. **Solution:** $\int e^{\tan^{-1} x} [1 + \frac{x}{1+x^2}] dx$. This is $e^{f(x)}(g'(x) + g(x)f'(x))$ form. Result: $x e^{\tan^{-1} x} + C$.

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