Unit III: Calculus - Differential Equations

SOLUTIONS

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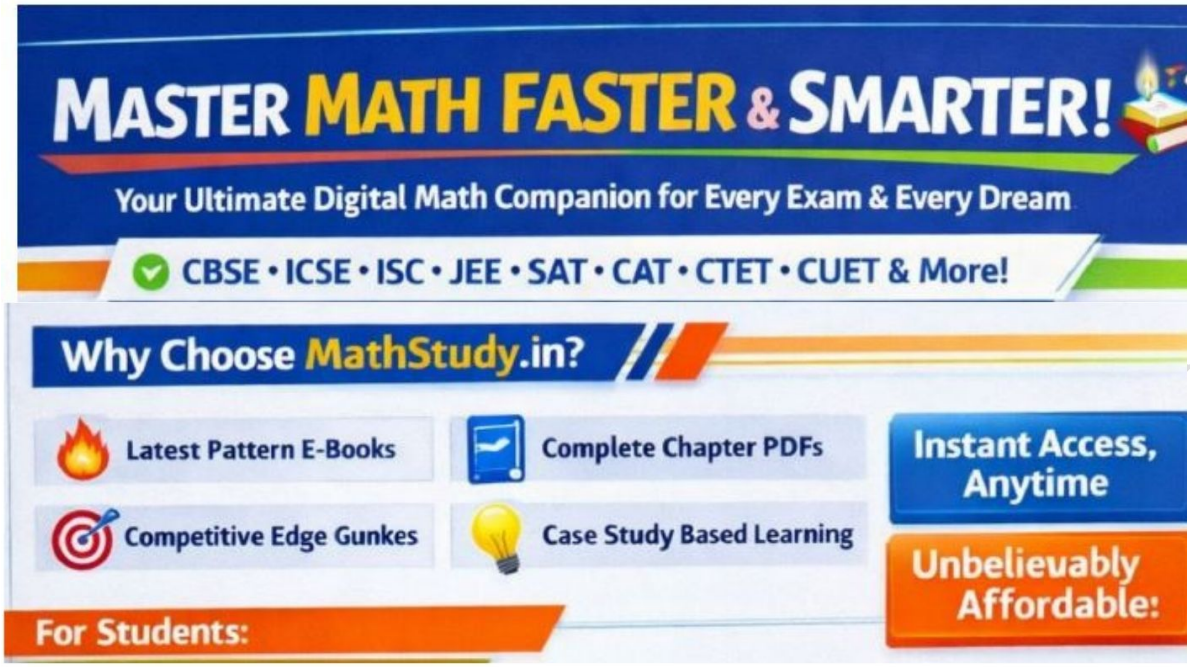

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Solutions

1. **Solution:** Let $x+y=v \Rightarrow 1+dy/dx=dv/dx$. Equation becomes $dv/dx-1=\frac{v+1}{2v+3} \Rightarrow dv/dx=\frac{3v+4}{2v+3}$. Integrate by parts/partial fractions. Result: $\frac{2}{3}(x+y)+\frac{1}{9}\log|3x+3y+4|=x+C$.

2. **Solution:** Bernoulli equation. Divide by y^2: $\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{xy}=\log x$. Let $1/y=z \Rightarrow -\frac{1}{y^2}\frac{dy}{dx}=dz/dx$. Result: $\frac{1}{y}=\log x+1+Cx$.

3. **Solution:** $\frac{dy}{dx}-\frac{y}{x}=x$. $IF=1/x$. $y/x=\int 1dx=x+C$. Since it passes through $(1,1)$, $C=0$. Result: $y=x^2$.

4. **Solution:** Put $y=vx$. $\int \frac{1+v}{1-v^2+v}dv=\int \frac{dx}{x}$. Result: $\log|x|+\frac{y}{x}-\log|1-y/x|=C$ (after simplification).

5. **Solution:** $IF=\sin x$. $y\sin x=\int 4x\cos x dx=2x^2+C$. At $x=\pi/2, y=0 \Rightarrow C=-\pi^2/2$. Result: $y\sin x=2x^2-\pi^2/2$.

6. **Solution:** $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$. $IF=e^{\tan^{-1}y}$. Result: $xe^{\tan^{-1}y}=e^{\tan^{-1}y}(\tan^{-1}y-1)+C$.

7. **Solution:** Let $x+y=v \Rightarrow dv/dx-1=\sin v+\cos v$. Integrate $\frac{dv}{1+\sin v+\cos v}=dx$. Result: $\log|1+\tan(\frac{x+y}{2})|=x+C$.

8. **Solution:** Homogeneous. $v+x\frac{dv}{dx}-v+\sin v=0 \Rightarrow \operatorname{cosec} v dv=-dx/x$. Result: $x\tan(\frac{y}{2x})=C$.

9. **Solution:** $2yy'=4a$. Replace y' with $-1/y'$. $2y(-1/y')=4a \Rightarrow -2ydx/dy=y^2/x \Rightarrow -2xdx=ydy$. Result: $2x^2+y^2=C$ (Family of ellipses).

10. **Solution:** $y=vx$. $v+x\frac{dv}{dx}=v+\sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x}$. Result: $y+\sqrt{x^2+y^2}=Cx^2$.

11. **Solution:** Linear in $(y-3)$. Let $y-3=Y, x+1=X$. $dY/dX=X+Y/X \Rightarrow dY/dX-Y/X=X$. $IF=1/X$. Result: $y-3=(x+1)^2+C(x+1)$. Use $(2,0)$ to find C.

12. **Solution:** Let $e^y=v \Rightarrow e^y\frac{dy}{dx}=\frac{dv}{dx}$. $\frac{dv}{dx}+\frac{v}{x}=\frac{1}{x^2}$. $IF=x$. $vx=\log x+C$. Result: $e^y x=\log x+C$.

13. **Solution:** $\frac{dx}{dy}+\frac{x}{y}=y^2$. $IF=y$. $xy=y^4/4+C$. Result: $4xy=y^4+C$.

14. **Solution:** Homogeneous. $v+x\frac{dv}{dx}=v^2+2v \Rightarrow \frac{dv}{v^2+v}=\frac{dx}{x}$. Result: $y=\frac{x}{1-x+1}$? Solve: $\frac{y}{x+y}=x$. Result: $y=\frac{x^2}{2-x}$.

15. **Solution:** $IF=1+x^3$. $y(1+x^3)=\int \sin^2 x dx=\frac{1}{2}(x-\frac{\sin 2x}{2})+C$.

16. **Solution:** $IF=y$. $xy=y^4/4+C$. Result: $x=\frac{y^3}{4}+\frac{C}{y}$.

17. **Solution:** Subtangent $=y/(dy/dx)=k$. $ydx=kdy \Rightarrow dx=k\frac{dy}{y}$. Result: $x=k\log y+C$.

18. **Solution:** $\frac{dx}{dy}-x=y+1$. $IF=e^{-y}$. $xe^{-y}=\int(y+1)e^{-y}dy$. Result: $x+y+2=Ce^y$.

19. **Solution:** Divide by x^2. Use $y=vx$. Result: $xy\cos(y/x)=C$.

20. **Solution:** Use the method of grouping terms or finding an Integrating Factor $\mu(y)$. Result: $xy^4+2xy+y^2=C$.

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