Unit III: Calculus - Continuity and Differentiability

General Instructions

1. Total Questions: **20**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Read each question carefully before answering.
5. Choose the most appropriate answer from the given options.
6. Use of calculator or electronic devices is strictly prohibited.

1. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x > 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, find the value of a.

2. Find the values of a and b such that the function $f(x) = \begin{cases} x^2 + 3x + a & x \leq 1 \\ bx + 2 & x > 1 \end{cases}$ is differentiable at $x = 1$.

3. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \infty}}}$, prove that $(2y - 1)\frac{dy}{dx} = \frac{1}{x}$.

4. Let $f(x)$ be a differentiable function satisfying $f(x + y) = f(x) + f(y) + xy^2 + x^2y$ for all real x, y. If $\lim_{h\to 0} \frac{f(h)}{h} = 1$, find $f'(x)$.

5. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find the value of $\frac{d^2y}{dx^2}$ in terms of t.

6. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ for $|x| < 1$.

7. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

8. Find the derivative of $f(x) = |x - 1| + |x + 1|$ at $x = 0$. Is it differentiable at $x = 1$?

9. If $y = (x + \sqrt{x^2 + 1})^m$, show that $(x^2 + 1)y_2 + xy_1 - m^2y = 0$.

10. Let $f(x) = [x^2]\sin(\pi x)$, where $[\cdot]$ denotes the greatest integer function. Find the points of discontinuity of $f(x)$ in the interval $(1, 2)$.

11. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, show that $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$.

12. If $f(x) = \log_e(\log_e x)$, find $f''(e)$.

13. Let $f(x + y) = f(x)f(y)$ for all x, y. If $f(5) = 2$ and $f'(0) = 3$, find the value of $f'(5)$.

14. If $y = \cos^{-1}\left(\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right)$, find dy/dx.

15. Prove that the function $f(x) = \begin{cases} x\sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous but not differentiable at $x = 0$.

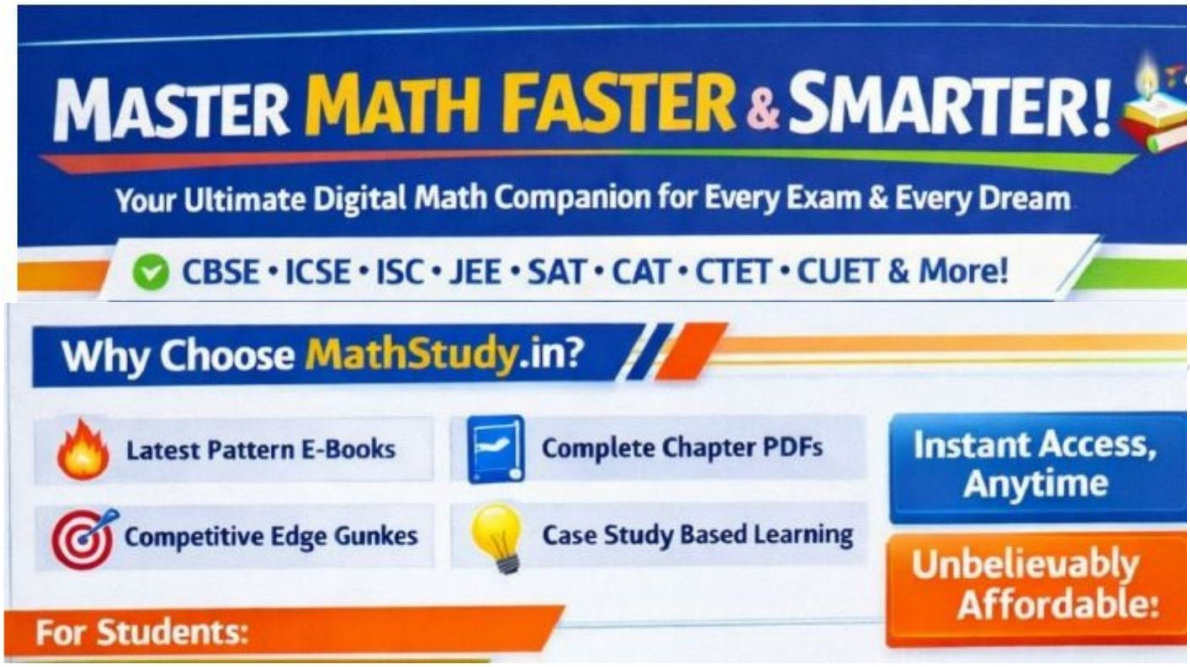
16. If $y = \tan^{-1} x$, show that $(1 + x^2)y_2 + 2xy_1 = 0$. Hence find y_3.

17. Determine the value of k so that $f(x) = \begin{cases} \frac{k\cos x}{\pi-2x} & x \neq \pi/2 \\ 3 & x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$.

18. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -e^{y-x}$.

19. Find the points where the function $f(x) = \max\{x, x^3\}$ is not differentiable.

20. If $y = \sin(\log x)$, prove that $x^2y_2 + xy_1 + y = 0$.



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