Unit III: Calculus - Applications of Derivatives

SOLUTIONS

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Solutions

1. **Solution:** $dy/dx = 3x^2 + 2$. Slope of normal $= -1/(3x^2 + 2)$. Parallel line slope $= -1/14$. Thus $3x^2 + 2 = 14 \Rightarrow x = \pm 2$. Points are $(2, 18)$ and $(-2, -6)$. Equations are $x + 14y - 254 = 0$ and $x + 14y + 86 = 0$.

2. **Solution:** $V = \pi r^2 h = 100\pi h$. $dV/dt = 100\pi(dh/dt)$. $314 = 100(3.14)(dh/dt) \Rightarrow dh/dt =$ 1 m/h.

3. **Solution:** $f'(x) = \frac{\cos x(2-\cos x)^2}{(2+\cos x)^2}$. For increasing, $\cos x > 0$. In $[0, 2\pi]$, this occurs in $(0, \pi/2)$ and $(3\pi/2, 2\pi)$.

4. **Solution:** Surface Area $S = \pi r^2 + \pi rl$. Express V in terms of r and S. Setting $dV/dr = 0$ and using $r = l \sin\alpha$ leads to $\sin\alpha = 1/3$.

5. **Solution:** $2y(dy/dt) = 8(dx/dt)$. Given $dy/dt = dx/dt \Rightarrow 2y = 8 \Rightarrow y = 4$. Then $16 = 8x \Rightarrow x = 2$. Point is $(2, 4)$.

6. **Solution:** $f'(x) = 6x^2 - 18ax + 12a^2 = 6(x - a)(x - 2a)$. Extremas at $a, 2a$. If $a > 0$, $p = a, q = 2a$. $p^2 = q \Rightarrow a^2 = 2a \Rightarrow a = 2$ (since $a \neq 0$).

7. **Solution:** Minimize $D^2 = (y^2 - x_L)^2 + (y - y_L)^2$. Alternatively, find point on $x = y^2$ where $dx/dy = 1$. $2y = 1 \Rightarrow y = 1/2, x = 1/4$. Distance from $(1/4, 1/2)$ to $x - y + 1 = 0$ is $\frac{|1/4-1/2+1|}{\sqrt{2}} = 3\sqrt{2}/8$.

8. **Solution:** $4s+2\pi r = 28$. $A = s^2+\pi r^2$. Minimize $A(r)$. $s = 28\pi/(4+\pi)$ and $r = 14/(4+\pi)$. Pieces are $112/(4 + \pi)$ and $28\pi/(4 + \pi)$.

9. **Solution:** Slopes $m_1 = 1/(2y)$ and $m_2 = -y/x$. Orthogonality $m_1 m_2 = -1 \Rightarrow 1/(2x) = 1 \Rightarrow x = 1/2$. Then $y^2 = 1/2$ and $(1/2)^2 y^2 = k^2 \Rightarrow 1/4(1/2) = k^2 \Rightarrow 8k^2 = 1$.

10. **Solution:** $f'(x) = \cos x - \sin 2x = \cos x(1 - 2\sin x)$. $x = \pi/6$. $f(0) = 1/2, f(\pi/6) = 3/4, f(\pi/2) = 1/2$. Max is $3/4$, Min is $1/2$.

11. **Solution:** $V = x^3 \Rightarrow dV/dt = 3x^2(dx/dt) = 9 \Rightarrow dx/dt = 3/x^2$. $S = 6x^2 \Rightarrow dS/dt = 12x(dx/dt) = 12x(3/x^2) = 36/x$. At $x = 10, dS/dt = 3.6$ sq cm/s.

12. **Solution:** Tangent: $y - (3x_1^2 + 4) = 6x_1(x - x_1)$. Passes through $(0, 0) \Rightarrow -3x_1^2 - 4 = -6x_1^2 \Rightarrow 3x_1^2 = 4 \Rightarrow x_1 = \pm 2/\sqrt{3}$.

13. **Solution:** For three roots, Max > 0 and Min < 0. $f'(x) = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$. $f(1) = k - 2, f(-1) = k + 2$. So $(k - 2)(k + 2) < 0 \Rightarrow k \in (-2, 2)$.

14. **Solution:** $r_{cyl} = r, h_{cyl} = h(1 - r/R)$. $V = \pi r^2 h(1 - r/R)$. Max at $r = 2R/3$. $V = \frac{4}{27}\pi R^2 h = \frac{4}{27}\pi h^3 \tan^2\alpha$.

15. **Solution:** $f'(x) = \frac{k-2}{(\sin x+\cos x)^2}$. For increasing, $k - 2 > 0 \Rightarrow k > 2$.

16. **Solution:** $f'(x) = 2\cos 2x - 1 = 0 \Rightarrow \cos 2x = 1/2 \Rightarrow x = \pm\pi/6$. Max at $\pi/6$, Min at $-\pi/6$.

17. **Solution:** $dy/dx = 3/(2\sqrt{3x - 2})$. Slope of line $= 2$. Set $3/(2\sqrt{3x - 2}) = 2 \Rightarrow \sqrt{3x - 2} = 3/4 \Rightarrow x = 41/48$.

18. **Solution:** Minimize $D^2 = (x-3)^2+(x^2+7-7)^2 = x^4+x^2-6x+9$. $f'(x) = 4x^3+2x-6 = 0$ at $x = 1$. Distance $= \sqrt{(1 - 3)^2 + (1^2)^2} = \sqrt{5}$.

19. **Solution:** Let $f(x) = x - \log(1+x)$. $f'(x) = 1 - 1/(1+x) = x/(1+x)$. For $x > 0, f'(x) > 0$. Since $f(0) = 0$, $f(x) > 0$ for all $x > 0$.

20. **Solution:** $V = \frac{1}{3}\pi R^2 H$. $R^2 = r^2 - (H - r)^2 = 2rH - H^2$. $V = \frac{1}{3}\pi(2rH^2 - H^3)$. $dV/dH = 0 \Rightarrow 4rH - 3H^2 = 0 \Rightarrow H = 4r/3$.