Unit III: Calculus - Applications of Derivatives

General Instructions

1. Total Questions: **20**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Read each question carefully before answering.
5. Choose the most appropriate answer from the given options.
6. Use of calculator or electronic devices is strictly prohibited.

1. Find the equation of the normal to the curve $y = x^3 + 2x + 6$ which is parallel to the line $x + 14y + 4 = 0$.

2. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic meters per hour. Find the rate at which the depth of the wheat is increasing (Use $\pi = 3.14$).

3. Determine the interval in which the function $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is strictly increasing in $[0, 2\pi]$.

4. Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}(1/3)$.

5. Find the coordinates of the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

6. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ has a local maximum at $x = p$ and local minimum at $x = q$ such that $p^2 = q$, find the value of a.

7. Find the shortest distance between the line $y - x = 1$ and the curve $x = y^2$.

8. A wire of length 28 m is to be cut into two pieces. One piece is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area is minimum?

9. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

10. Find the absolute maximum and minimum values of the function $f(x) = \sin x + \frac{1}{2}\cos 2x$ in the interval $[0, \pi/2]$.

11. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 cm?

12. Find the point on the curve $y = 3x^2 + 4$ at which the tangent passes through the origin.

13. Let $f(x) = x^3 - 3x + k$. Find the range of k for which the equation $f(x) = 0$ has three distinct real roots.

14. Find the maximum volume of a cylinder which can be inscribed in a cone of height h and semi-vertical angle α.

15. If the function $f(x) = \frac{k\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all x, find the range of k.

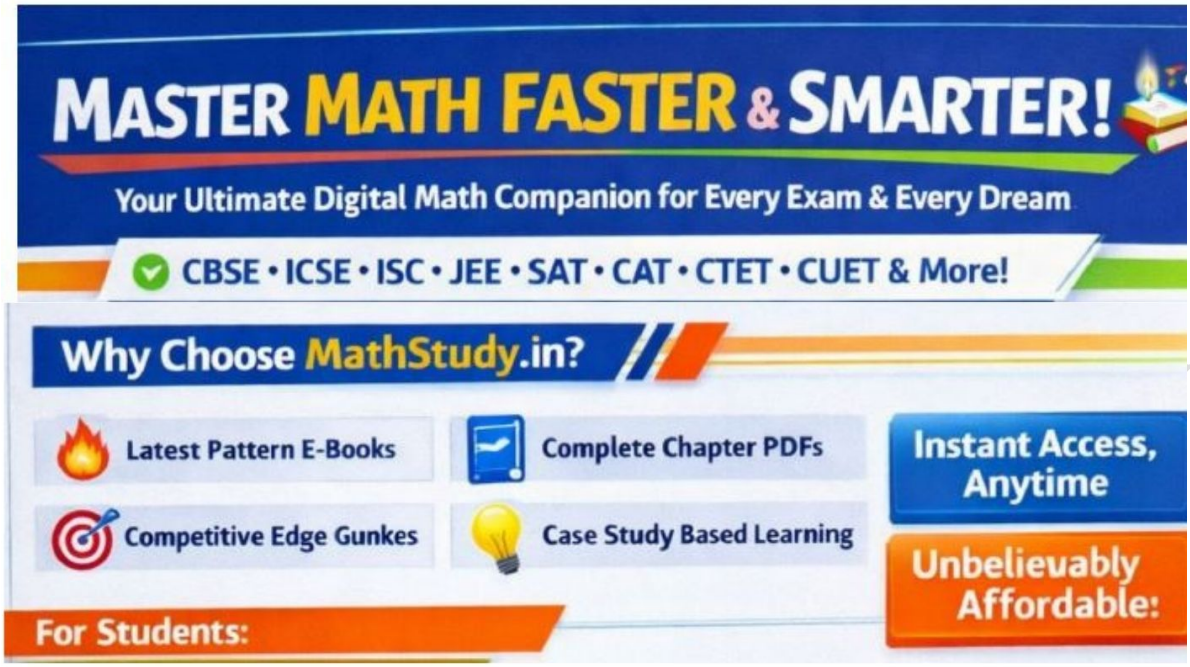
16. Determine the points of local maxima and local minima, if any, for the function $f(x) = \sin 2x - x$ in $(-\pi/2, \pi/2)$.

17. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

18. A jet of an enemy is flying along the curve $y = x^2 + 7$. A soldier, placed at $(3, 7)$, wants to shoot down the jet when it is nearest to him. Find the nearest distance.

19. Show that for all $x > 0$, the inequality $\log(1 + x) < x$ holds.

20. Find the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r.



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