# CUET Mathematics Test - Set 19

## Chapter: Applications of Integrals (Intermediate)

**SOLUTIONS**

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## Solutions

1. **Solution:** Intersection points: $x^2 = 4x \Rightarrow x = 0, 4$. Area $= \int_0^4 (2\sqrt{x} - x)dx = [\frac{4}{3}x^{3/2} - \frac{x^2}{2}]_0^4 = \frac{32}{3} - 8 = 8/3$. **Correct Option: (C)**

2. **Solution:** Area $= \int_0^{\pi/4} (\cos x - \sin x)dx = [\sin x + \cos x]_0^{\pi/4} = (1/\sqrt{2} + 1/\sqrt{2}) - (0 + 1) = \sqrt{2} - 1$. **Correct Option: (A)**

3. **Solution:** Intersection: $2 - x^2 = -x \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2, -1$. Area $= \int_{-1}^2 (2 - x^2 + x)dx = [2x - \frac{x^3}{3} + \frac{x^2}{2}]_{-1}^2 = 9/2$. **Correct Option: (A)**

4. **Solution:** Curves intersect at $x^2 + 1 = x + 1 \Rightarrow x = 0, 1$. From 0 to 1, $y = x^2 + 1$ is lower than $y = x + 1$. Area $= \int_0^1 (x^2 + 1)dx + \int_1^2 (x + 1)dx$ based on inequality constraints. 19/6. **Correct Option: (D)**

5. **Solution:** Standard result for $y^2 = 4ax, x^2 = 4by$ is Area $= \frac{16ab}{3}$. Here $a = 1, b = 1$, so 16/3. **Correct Option: (A)**

6. **Solution:** Area $= \int_{-2}^{-1} -(x+1)dx + \int_{-1}^3 (x+1)dx = [-\frac{x^2}{2} - x]_{-2}^{-1} + [\frac{x^2}{2} + x]_{-1}^3 = 1/2 + 8 = 17/2$. **Correct Option: (C)**

7. **Solution:** This is a semi-circle with radius 4. Area $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(16) = 8\pi$. **Correct Option: (B)**

8. **Solution:** Area $= \int_{-1}^0 -x(-x)dx + \int_0^1 x(x)dx$ is incorrect. $y = -x^2$ for $x < 0$ and $x^2$ for $x > 0$. Area $= |\int_{-1}^0 -x^2 dx| + \int_0^1 x^2 dx = 1/3 + 1/3 = 2/3$. **Correct Option: (B)**

9. **Solution:** Using Area formula or Integration of lines $AB, BC, CA$: Area $= 1/2|1(2-1) + 2(1-0) + 3(0-2)| = 1/2|1 + 2 - 6| = 3/2$. **Correct Option: (A)**

10. **Solution:** Symmetry: $2\int_0^1 (x - x^2)dx = 2[\frac{x^2}{2} - \frac{x^3}{3}]_0^1 = 2(1/6) = 1/3$. **Correct Option: (B)**

11. **Solution:** $4y = x^2$ and $4y = x + 2$. $x^2 = x + 2 \Rightarrow x = 2, -1$. Area $= \int_{-1}^2 (\frac{x+2}{4} - \frac{x^2}{4})dx = \frac{1}{4}[\frac{x^2}{2} + 2x - \frac{x^3}{3}]_{-1}^2 = 9/8$. **Correct Option: (A)**

12. **Solution:** Loop exists for $x \in [0, 1]$. $y = \pm\sqrt{x}(1 - x)$. Area $= 2\int_0^1 (\sqrt{x} - x^{3/2})dx = 2[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}]_0^1 = 2(4/15) = 8/15$. **Correct Option: (B)**

13. **Solution:** Intersects x-axis at $x = 1 - e$. Area $= \int_{1-e}^0 \log(x + e)dx$. Let $x + e = t$. $[t \log t - t]_1^e = (e - e) - (0 - 1) = 1$. **Correct Option: (A)**

14. **Solution:** Intersection at $x^2 = 2/(1 + x^2) \Rightarrow x^4 + x^2 - 2 = 0 \Rightarrow x^2 = 1$. $x = \pm 1$. Area $= 2\int_0^1 (\frac{2}{1+x^2} - x^2)dx = 2[2\tan^{-1} x - \frac{x^3}{3}]_0^1 = 2(\pi/2 - 1/3) = \pi - 2/3$. **Correct Option: (B)**

15. **Solution:** Area $= \int_0^{\pi/4} \tan x dx = [\log|\sec x|]_0^{\pi/4} = \log\sqrt{2} - \log 1 = \log\sqrt{2}$. **Correct Option: (A)**