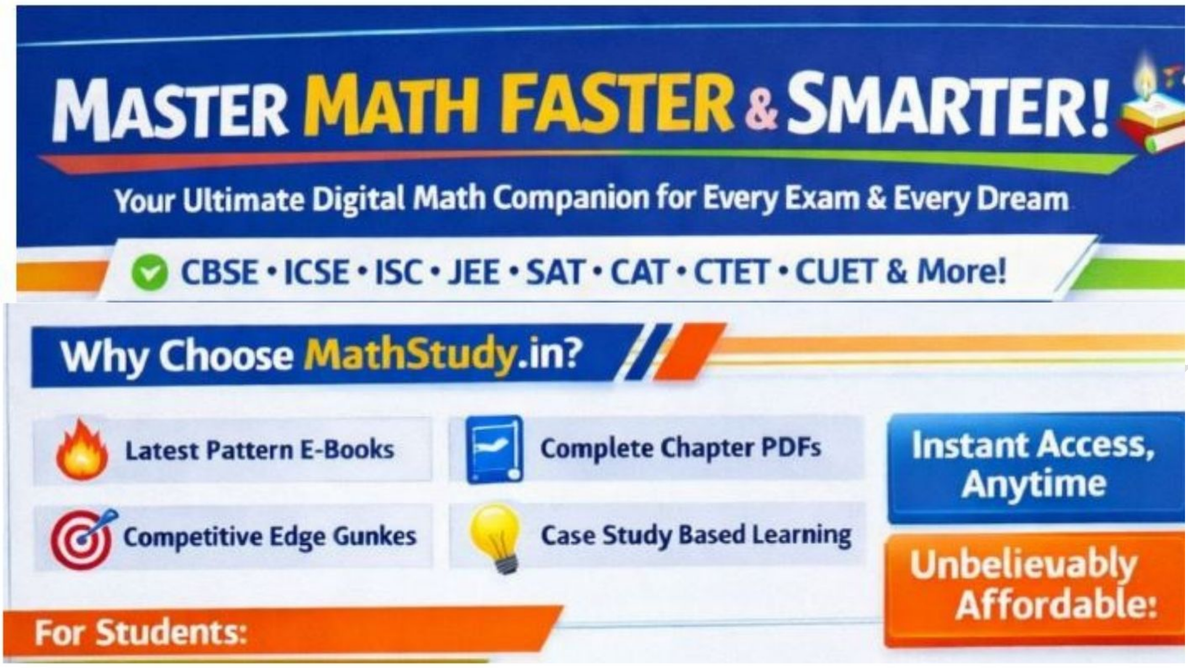
Unit III: Calculus - Applications of Integrals

SOLUTIONS



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Solutions

1. **Solution:** By symmetry across both axes, calculate area in the first quadrant for $y = |\ln x|$ from $x = 1/e$ to e. Area $= 4[\int_{1/e}^{1} -\ln x dx + \int_1^e \ln x dx]$. Result: $4(e + 1/e - 2)$.

2. **Solution:** This is the segment of the circle $x^2 + y^2 = 1$ cut by the line $x + y = 1$. Area $= \frac{\pi(1)^2}{4} - \frac{1}{2}(1)(1) = \frac{\pi-2}{4}$.

3. **Solution:** Intersections at $x = 1/2$. Area $= 2[\int_0^{1/2} \sqrt{4x}dx + \int_{1/2}^{3/2} \sqrt{9/4 - x^2}dx]$. Result: $\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}(1/3)$.

4. **Solution:** Symmetry about y-axis. Area $= 2\int_0^1 (x - x^2)dx = 2[1/2 - 1/3] = 1/3$.

5. **Solution:** Intersections: $(4x-1)^2 = 2x \Rightarrow 16x^2 - 10x + 1 = 0 \Rightarrow x = 1/2, 1/8$. Area $= \int_{1/8}^{1/2} (\sqrt{2x} - (4x-1))dx$. Result: $9/32$.

6. **Solution:** Intersection of $y = \sqrt{x}$ and $y = (x-3)/2$ is at $x = 9, y = 3$. Area $= \int_0^9 \sqrt{x}dx - \int_3^9 \frac{x-3}{2}dx = 18 - 9 = 9$.

7. **Solution:** Curves intersect at $x = \pi/4$. Area $= \int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx = 2(\sqrt{2} - 1)$.

8. **Solution:** Area $= \int_0^1 (x^2+1)dx + \int_1^2 (x+1)dx = [x^3/3+x]_0^1 + [x^2/2+x]_1^2 = 4/3 + 5/2 = 23/6$.

9. **Solution:** Identical to Q4. Area $= 1/3$.

10. **Solution:** Tangent: $x + \sqrt{3}y = 4$. Normal: $y - \sqrt{3} = \sqrt{3}(x-1)$. Tangent meets x-axis at $(4, 0)$, Normal at $(0, 0)$. Area of triangle with vertices $(0, 0), (1, \sqrt{3}), (4, 0)$ is $1/2 \times 4 \times \sqrt{3} = 2\sqrt{3}$.

11. **Solution:** Intersection points $(1, 2)$ and $(-1, 2)$ and $(2, 1)$ and $(-2, 1)$. Area is a square/diamond. Result: 4.

12. **Solution:** Area $= \int_0^4 (4 - \sqrt{x})dx = [4x - \frac{2}{3}x^{3/2}]_0^4 = 16 - 16/3 = 32/3$.

13. **Solution:** Tangent at $(1, 1)$ is $y - 1 = 3(x-1) \Rightarrow y = 3x - 2$. Intersects x-axis at $2/3$. Area $= \int_0^1 x^3 dx -$ Area of triangle $= 1/4 - (1/2 \times 1/3 \times 1) = 1/4 - 1/6 = 1/12$.

14. **Solution:** Intersections at $x = 1$ and $x = -4$. Area $= \int_{-4}^{1} (\frac{x+9}{5} - \sqrt{|x+3|})dx$. Result: $3/2 - 4/3 = 1/6$.

15. **Solution:** Area $= \int_0^1 (e^x - e^{-x})dx = [e^x + e^{-x}]_0^1 = e + 1/e - 2$.

16. **Solution:** Put $x^2 = t$. Area $= \int_0^{\pi/2} \frac{1}{2}\sin t dt = [-\frac{1}{2}\cos t]_0^{\pi/2} = 1/2$.

17. **Solution:** Circle $(x-4)^2 + y^2 = 16$. Intersects $y^2 = 4x$ at $x = 0, 4$. Area $= \int_0^4 (\sqrt{16 - (x-4)^2} - \sqrt{4x})dx$. Result: $2\pi - 8/3$.

18. **Solution:** First quadrant area. Area $= \int_0^2 (\frac{1}{2}\sqrt{4 - x^2} - (1 - x/2))dx = \pi/2 - 1$.

19. **Solution:** Symmetry. $2\int_0^1 (x - x^2)dx = 1/3$.

20. **Solution:** Tangent at $x = \pi/4$ is $y - 1 = 2(x - \pi/4)$. Intersects x-axis at $x = \pi/4 - 1/2$. Area $= \int_0^{\pi/4} \tan x dx -$ Triangle Area $= \ln\sqrt{2} - 1/4$.