CUET Mathematics Test - Set 17

Chapter: Calculus - Applications of Derivatives (Intermediate)

SOLUTIONS

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1. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
(A) 8/3 cm/s
(B) 4/3 cm/s
(C) 2/3 cm/s
(D) 1 cm/s

2. The function $f(x) = \frac{x}{\log x}$ is increasing in the interval:
(A) $(0, 1)$
(B) $(1, e)$
(C) (e, ∞)
(D) $(-\infty, e)$

3. The angle of intersection of the curves $y^2 = x$ and $x^2 = y$ at $(1, 1)$ is:
(A) $\tan^{-1}(3/4)$
(B) $\tan^{-1}(4/3)$
(C) $\pi/2$
(D) $\pi/4$

4. The sum of two non-negative numbers is 20. If the product of the square of one and the cube of the other is maximum, the numbers are:
(A) 10, 10
(B) 12, 8
(C) 8, 12
(D) 15, 5

5. The value of x for which the function $f(x) = x(x-2)^2$ is maximum in its domain is:
(A) 2
(B) 2/3
(C) 1
(D) 0

6. The rate of change of the volume of a sphere with respect to its surface area when the radius is 2 units is:
(A) 1
(B) 2
(C) 3
(D) 4

7. The equation of the normal to the curve $y = \sin x$ at $(0, 0)$ is:
(A) $x + y = 0$
(B) $x - y = 0$
(C) $y = 0$
(D) $x = 0$

8. If the subnormal to the curve $xy^n = a^{n+1}$ is constant, then n is:
(A) 1
(B) 2
(C) -2
(D) -1

9. For the curve $y = x^2 + 3x + 4$, the tangent at the point P passes through the origin. The coordinates of P are:

(A) $(2, 14)$ and $(-2, 2)$
(B) $(2, 14)$ and $(1, 8)$
(C) $(0, 4)$
(D) $(2, 14)$ only

10. The maximum value of $f(x) = \frac{\log x}{x}$ for $x > 0$ is:
(A) e
(B) $1/e$
(C) e^2
(D) 1

11. The interval in which the function $f(x) = \sin x - \cos x$ is decreasing for $0 < x < 2\pi$ is:
(A) $(0, 3\pi/4)$
(B) $(3\pi/4, 7\pi/4)$
(C) $(\pi/4, 5\pi/4)$
(D) $(0, \pi)$

12. The point on the curve $y = x^2 - 4x + 3$ where the tangent is parallel to the chord joining $(1, 0)$ and $(4, 3)$ is:
(A) $(2, -1)$
(B) $(5/2, -3/4)$
(C) $(3, 0)$
(D) $(1, 0)$

13. The volume of the largest cone that can be inscribed in a sphere of radius R is:
(A) $\frac{8}{27}$ (Volume of sphere)
(B) $\frac{1}{2}$ (Volume of sphere)
(C) $\frac{1}{3}$ (Volume of sphere)
(D) $\frac{4}{9}$ (Volume of sphere)

14. If $f(x) = x^3 + ax^2 + bx + c$ has a local maximum at $x = -1$ and local minimum at $x = 3$, then:
(A) $a = -3, b = -9$
(B) $a = 3, b = 9$
(C) $a = -3, b = 9$
(D) $a = 3, b = -9$

15. Water is dripping out from a conical funnel of semi-vertical angle $\pi/4$ at the rate of 2 cu. cm/sec. When the slant height of water is 4 cm, the rate of decrease of the slant height is:
(A) $\frac{\sqrt{2}}{4\pi}$ cm/s
(B) $\frac{1}{4\pi\sqrt{2}}$ cm/s
(C) $\frac{\sqrt{2}}{8\pi}$ cm/s
(D) $\frac{1}{8\pi}$ cm/s

Solutions

1. **Solution:** $x^2 + y^2 = 25$. Differentiating w.r.t. t: $2x(dx/dt) + 2y(dy/dt) = 0$. Given $x = 4$, then $y = 3$. $2(4)(2) + 2(3)(dy/dt) = 0 \Rightarrow 16 + 6(dy/dt) = 0 \Rightarrow dy/dt = -8/3$. **Correct Option: (A)**

2. **Solution:** $f'(x) = \frac{\log x - 1}{(\log x)^2}$. For increasing, $f'(x) > 0 \Rightarrow \log x > 1 \Rightarrow x > e$. **Correct Option: (C)**

3. **Solution:** $y^2 = x \Rightarrow 2yy' = 1 \Rightarrow m_1 = 1/2$. $x^2 = y \Rightarrow 2x = y' \Rightarrow m_2 = 2$. $\tan\theta = |\frac{2-1/2}{1+2(1/2)}| = |\frac{3/2}{2}| = 3/4$. **Correct Option: (A)**

4. **Solution:** $x + y = 20$. Maximize $P = x^2y^3 = x^2(20-x)^3$. $dP/dx = 2x(20-x)^3 - 3x^2(20-x)^2 = x(20-x)^2[40 - 2x - 3x] = x(20-x)^2(40-5x)$. $x = 8$. Then $y = 12$. **Correct Option: (C)**

5. **Solution:** $f'(x) = (x-2)^2 + 2x(x-2) = (x-2)(x-2+2x) = (x-2)(3x-2)$. Maxima at $x = 2/3$. **Correct Option: (B)**

6. **Solution:** $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$. $dV/dr = 4\pi r^2$, $dS/dr = 8\pi r$. $dV/dS = \frac{4\pi r^2}{8\pi r} = r/2$. At $r = 2$, $dV/dS = 1$. **Correct Option: (A)**

7. **Solution:** $dy/dx = \cos x$. At $(0,0)$, slope of tangent $m = 1$. Slope of normal $= -1$. Equation: $y - 0 = -1(x - 0) \Rightarrow x + y = 0$. **Correct Option: (A)**

8. **Solution:** Length of subnormal is $y(dy/dx)$. $xy^n = a^{n+1} \Rightarrow y^n + nxy^{n-1}y' = 0 \Rightarrow y' = -y/nx$. Subnormal $= -y^2/nx$. For this to be constant, $y^2 \propto x$, which happens when $n = -2$. **Correct Option: (C)**

9. **Solution:** Let P be (x_1, y_1). Tangent: $y - y_1 = (2x_1 + 3)(x - x_1)$. Since it passes through $(0,0)$: $-y_1 = -2x_1^2 - 3x_1$. Also $y_1 = x_1^2 + 3x_1 + 4$. Solving gives $x_1^2 = 4 \Rightarrow x_1 = \pm 2$. Points are $(2, 14)$ and $(-2, 2)$. **Correct Option: (A)**

10. **Solution:** $f'(x) = \frac{1 - \log x}{x^2} = 0 \Rightarrow x = e$. $f(e) = \log e/e = 1/e$. **Correct Option: (B)**

11. **Solution:** $f'(x) = \cos x + \sin x$. For decreasing, $f'(x) < 0 \Rightarrow \tan x < -1$. In $(0, 2\pi)$, this occurs in $(3\pi/4, 7\pi/4)$ and $(\pi/2, 3\pi/4)$ etc. The most suitable interval provided is $3\pi/4$ to $7\pi/4$. **Correct Option: (B)**

12. **Solution:** Slope of chord $= (3-0)/(4-1) = 1$. $f'(x) = 2x - 4$. Set $2x - 4 = 1 \Rightarrow x = 5/2$. $y = (25/4) - 10 + 3 = -3/4$. **Correct Option: (B)**

13. **Solution:** Volume of largest cone is $\frac{8}{27}$ of the volume of the sphere (standard result from optimization). **Correct Option: (A)**

14. **Solution:** $f'(x) = 3x^2 + 2ax + b$. Roots are -1 and 3. Sum of roots: $-2a/3 = 2 \Rightarrow a = -3$. Product of roots: $b/3 = -3 \Rightarrow b = -9$. **Correct Option: (A)**

15. **Solution:** $r = l\sin(\pi/4) = l/\sqrt{2}$, $h = l/\sqrt{2}$. $V = \frac{1}{3}\pi r^2 h = \frac{\pi l^3}{6\sqrt{2}}$. $dV/dt = \frac{\pi l^2}{2\sqrt{2}}(dl/dt)$. $2 = \frac{\pi(16)}{2\sqrt{2}}(dl/dt) \Rightarrow dl/dt = \frac{\sqrt{2}}{4\pi}$. **Correct Option: (A)**