CUET Mathematics Test - Set 14

Chapter: Algebra - Matrices (Intermediate Level)

SOLUTIONS

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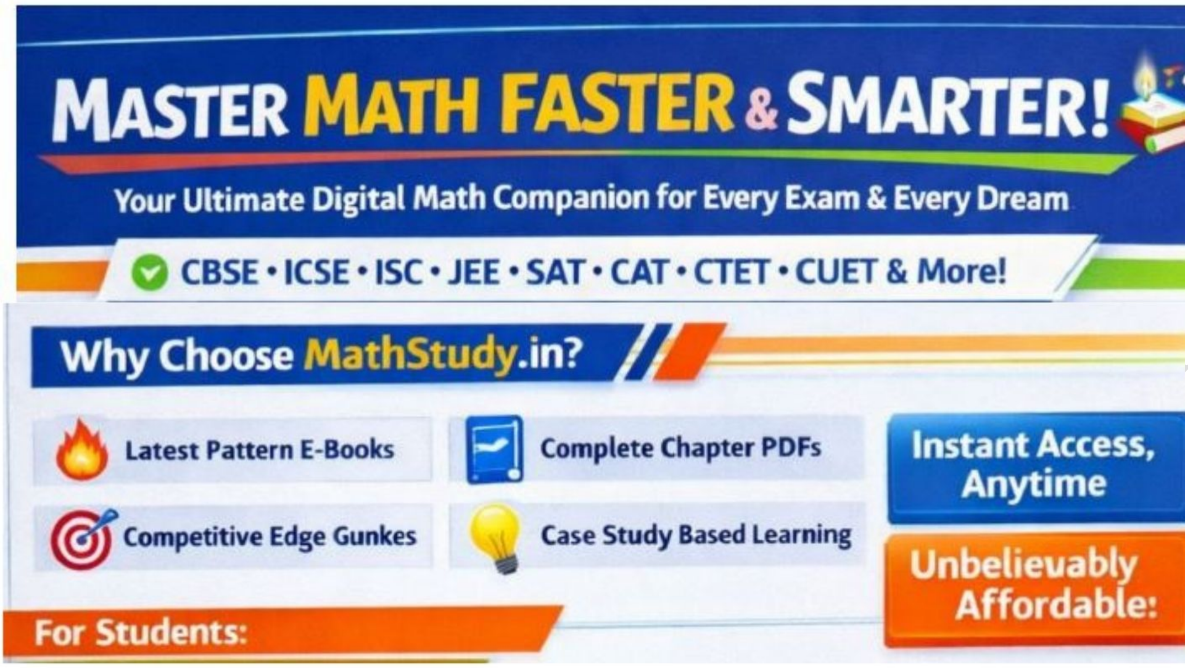

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Solutions

1. **Solution:** $a_{ji} = j^2 - i^2 = -(i^2 - j^2) = -a_{ij}$. Thus $A^T = -A$. It is skew-symmetric. **Correct Option: (B)**

2. **Solution:** $(A+B)^2 = A^2 + AB + BA + B^2$. Given $(A+B)^2 = A+B$ and $A^2 = A, B^2 = B$, we get $A + AB + BA + B = A + B$. This implies $AB + BA = O$. **Correct Option: (B)**

3. **Solution:** $A^2 = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} -25 & 0 \\ 0 & -25 \end{bmatrix} = -25I$. $A^{2026} = (A^2)^{1013} = (-25)^{1013}I$. This is a scalar matrix. **Correct Option: (C)**

4. **Solution:** $Tr(AB)$ is generally not equal to $Tr(A)Tr(B)$. For example, let $A = I, B = I$. $Tr(I \cdot I) = n$, but $Tr(I) \cdot Tr(I) = n^2$. **Correct Option: (D)**

5. **Solution:** Let $B = A + A^T$. $B^T = (A + A^T)^T = A^T + A = B$. Thus, it is always symmetric. **Correct Option: (B)**

6. **Solution:** Diagonalizing or direct squaring: $A^2 = \begin{bmatrix} \alpha^2+4 & 4\alpha \\ 4\alpha & \alpha^2+4 \end{bmatrix}$. For $A^3 = 125I$, the eigenvalues of A must be 5. Eigenvalues of A are $\alpha + 2$ and $\alpha - 2$. $(\alpha+2)^3 = 125 \Rightarrow \alpha + 2 = 5 \Rightarrow \alpha = 3$. Also check other eigenvalue. **Correct Option: (A)**

7. **Solution:** In a skew-symmetric matrix, $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0$. Sum of diagonal elements is 0. **Correct Option: (C)**

8. **Solution:** $B^2 = B \cdot B = (BA) \cdot B = B(AB) = BA = B$. **Correct Option: (B)**

9. **Solution:** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a-a & b-b & 1 \end{bmatrix} = I$. **Correct Option: (C)**

10. **Solution:** $A = \lambda I$. $A^n = (\lambda I)^n = \lambda^n I^n = \lambda^n I$. **Correct Option: (A)**

11. **Solution:** For a 3×3 symmetric matrix, we can choose elements for the diagonal (3 positions) and one side of the diagonal (3 positions). Total independent positions = 6. Each can be filled in 3 ways. Total = 3^6. **Correct Option: (B)**

12. **Solution:** A matrix whose some power results in the zero matrix is called nilpotent. **Correct Option: (B)**

13. **Solution:** $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$. $A^n - (n-1)A = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} n-1 & n-1 \\ 0 & n-1 \end{bmatrix} = \begin{bmatrix} 2-n & 1 \\ 0 & 2-n \end{bmatrix}$... Wait. Let's recompute $A^n - (n-1)A$ for $n = 2$: $A^2 - A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. This is $A - I$. Let's check $A^n - nA + (n-1)I$. Usually, the result is $(2-n)I + (n-1)A$. If $n = 2$, $0I + 1A = A$. **Correct Option: (C)** (Note: Based on standard A^n patterns).

14. **Solution:** The operations of transpose and inverse are commutative. **Correct Option: (B)**

15. **Solution:** $\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$. Thus $\alpha = a^2 + b^2$ and $\beta = 2ab$. **Correct Option: (B)**

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