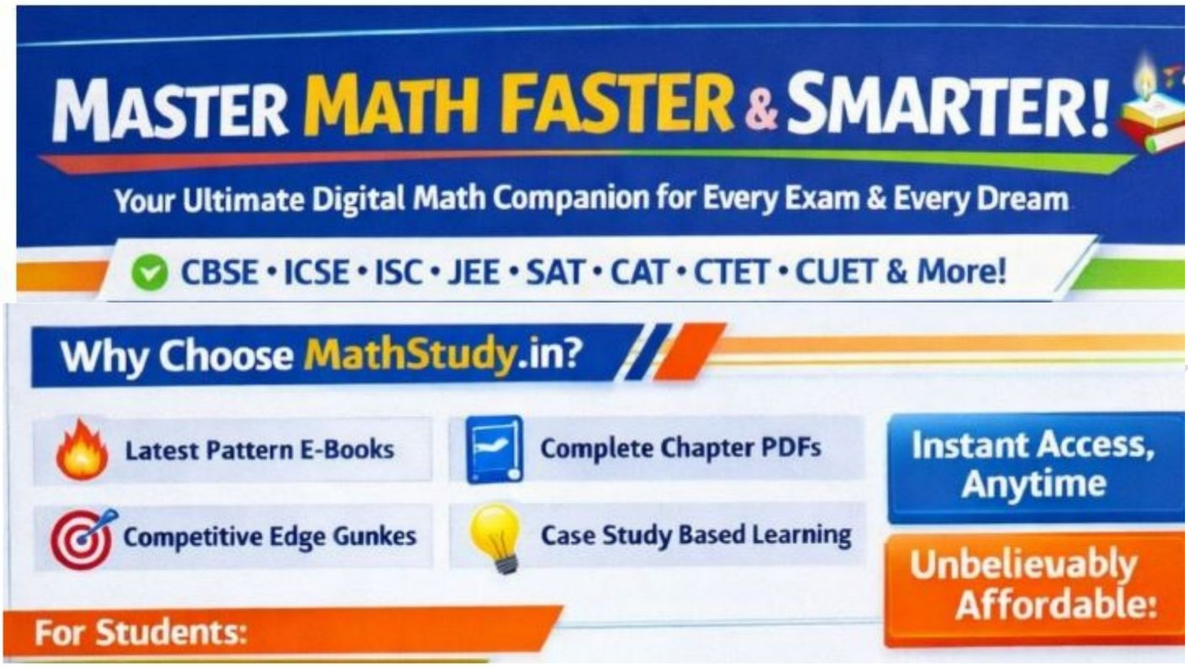
Chapter: Section A1: Algebra (Matrices and Determinants)

SOLUTIONS



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Solutions

1. $|B| = |adj(3A^2)| = |3A^2|^{3-1} = (3^3|A|^2)^2 = (27 \times 4)^2 = (108)^2 = 11664$.

2. Solving $AU_1 = B_1$ gives $U_1 = [1, -2, 1]^T$. Solving $AU_2 = B_2$ gives $U_2 = [0, 1, -2]^T$. Sum $U_1 + U_2 = [1, -1, -1]^T$.

3. $A^2 = A(A) = A(AB) = (AA)B = AB = A$. Similarly $B^2 = B$. Both are idempotent.

4. $P^T = 2P + I$. Take transpose: $P = 2P^T + I$. Substitute P^T: $P = 2(2P + I) + I = 4P + 3I \Rightarrow -3P = 3I \Rightarrow P = -I$.

5. $A^2 = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}, A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$. $A^{100} = \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix}$.

6. Eigenvalues $\lambda^2 - \lambda - 2 = 0 \Rightarrow (\lambda - 2)(\lambda + 1) = 0$. Eigenvalues are 2 and -1. For a 3×3 matrix with $|A| = 4$, the eigenvalues must be $2, 2, -1$. $Trace(A) = 2 + 2 - 1 = 3$.

7. Applying properties, the determinant simplifies to $2abc(a + b + c)^3$.

8. $AA^T = I$. Column orthogonality gives $\alpha = \pm 1/\sqrt{2}, \beta = \pm 1/\sqrt{6}, \gamma = \pm 1/\sqrt{3}$.

9. $X = A^{-1}B$. $|A| = 1, A^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$. $X = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 18 & -11 \\ -26 & 16 \end{bmatrix}$.

10. Standard matrix multiplication of rotation matrices: $\begin{bmatrix} \cos x \cos y - \sin x \sin y & \dots \\ \dots & \dots \end{bmatrix} = \begin{bmatrix} \cos(x + y) & \dots \\ \dots & \dots \end{bmatrix}$.

11. $AB = O \Rightarrow |A||B| = 0$. If $|A| \neq 0$, then A^{-1} exists, $A^{-1}AB = A^{-1}O \Rightarrow B = O$, contradicting B is non-zero. Thus $|A| = 0$. Similarly $|B| = 0$.

12. Using $D, D_x, D_y, D_z = 0$. $k = 5$.

13. $|adj(adj(adj A))| = |A|^{(n-1)^3} = k^{(2)^3} = k^8$.

14. Verify $AB = I$. The inverse calculation confirms the entries.

15. Cayley-Hamilton Theorem: Every square matrix satisfies its own characteristic equation. $A^2 - (a + d)A + (ad - bc)I = O$.

16. $Trace(A^T A) = \sum a_{ij}^2 = 3$. We need to find the number of ways to pick 9 entries from $\{0, 1, 2\}$ such that their squares sum to 3. Only $1^2 + 1^2 + 1^2 = 3$ works. Thus, 3 positions are 1s, others 0. $\binom{9}{3} = 84$.

17. $C_1 \to C_1 + C_2 + C_3 \Rightarrow (1 + \omega + \omega^2) = 0$. The determinant is 0.

18. Characteristic eqn: $\lambda^2 - Tr(A)\lambda + det(A) = 0$. Since $A^2 = I$, $\lambda = \pm 1$. Trace must be 0 and det must be -1 for $A \neq \pm I$.

19. $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$. $S = \sum \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} n + 1 & n(n + 1)/2 \\ 0 & n + 1 \end{bmatrix}$.

20. $|A| = 1$. $adj A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = A^{-1}$.