CUET Mathematics Test - Set 20

Chapter: Differential Equations (Intermediate)

General Instructions

1. Total Questions: **15**
2. Duration: **60 Minutes**
3. All questions are compulsory.
4. Each question carries **5 marks**.
5. For each correct answer: **+5 marks**.
6. For each incorrect answer: **-1 mark**.
7. No negative marking for unanswered questions.
8. Use of calculator or electronic devices is strictly prohibited.
9. Choose the most appropriate answer from the given options.

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1. The solution of the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$ is:
   (A) $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + C$
   (B) $\tan^{-1}\left(\frac{4x+y+1}{2}\right) = 2x + C$
   (C) $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = 2x + C$
   (D) None of these

2. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:
   (A) e^{-x}
   (B) e^{x}
   (C) $1/x$
   (D) x

3. The solution of $ydx + (x + x^2y)dy = 0$ is:
   (A) $-\frac{1}{xy} + \log y = C$
   (B) $\frac{1}{xy} + \log y = C$
   (C) $\frac{1}{xy} + \log x = C$
   (D) $-\frac{1}{xy} + \log x = C$

4. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$ is:
   (A) 1
   (B) 2
   (C) 3
   (D) Not defined

5. Find the general solution of $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$:
   (A) $x^2 + y^2 = Cx$
   (B) $x^2 - y^2 = Cx$
   (C) $x^2 + y^2 = Cy$
   (D) $x^2 - y^2 = Cy$

6. The particular solution of $\frac{dy}{dx} = y\tan x$, given $y = 1$ when $x = 0$, is:
   (A) $y = \cos x$
   (B) $y = \sec x$
   (C) $y = \sin x$
   (D) $y = \tan x$

7. The differential equation of the family of curves $y = a\cos(x + b)$ is:
   (A) $\frac{d^2y}{dx^2} + y = 0$
   (B) $\frac{d^2y}{dx^2} - y = 0$
   (C) $\frac{dy}{dx} + y = 0$
   (D) $\frac{d^2y}{dx^2} + a^2y = 0$

8. The solution of $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{1}{x}$ is:
   (A) $y\log x = \frac{(\log x)^2}{2} + C$
   (B) $y = \log x + C$
   (C) $y\log x = x\log x + C$
   (D) $y = \frac{\log x}{2} + C$

9. Which substitution will transform $(x - y)\frac{dy}{dx} = x + 2y$ into a separable form?
   (A) $x = v/y$
   (B) $y = vx$

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(C) $x + y = v$
(D) $x - y = v$

10. The general solution of $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is:
(A) $y - x + \log|x + y| = C$
(B) $y - x = \log|x + y| + C$
(C) $y - x + C = 0$
(D) $y - x = \log|x + y + 1| + C$

11. If $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only, then $y \cdot (IF)$ is equal to:
(A) $\int Q \cdot (IF)dx + C$
(B) $\int P \cdot (IF)dx + C$
(C) $\int Qdx + C$
(D) $Q \cdot (IF) + C$

12. The order of the differential equation whose general solution is $y = C_1e^x + C_2e^{2x} + C_3e^{x+C_4}$ is:
(A) 4
(B) 3
(C) 2
(D) 1

13. The solution of $\frac{dy}{dx} = \sqrt{4 - y^2}$ $(-2 < y < 2)$ is:
(A) $\cos^{-1}(y/2) = x + C$
(B) $\sin^{-1}(y/2) = x + C$
(C) $\sin^{-1} y = 2x + C$
(D) $y = 2\sin x + C$

14. The general solution of $(x + 2y^3)\frac{dy}{dx} = y$ is:
(A) $x = y^3 + Cy$
(B) $x = y^3 + C/y$
(C) $y = x^3 + Cx$
(D) $x = y^2 + Cy$

15. The area bounded by a curve is represented by a differential equation. If the slope of the tangent at any point (x, y) is x/y, the curve is:
(A) A circle
(B) A parabola
(C) A hyperbola
(D) An ellipse

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Solutions

1. **Solution:** Let $4x+y+1=v \implies 4+dy/dx=dv/dx \implies dy/dx=dv/dx-4$. Equation becomes $dv/dx-4=v^2 \implies \int \frac{dv}{v^2+4}=\int dx \implies \frac{1}{2}\tan^{-1}(v/2)=x+C$. **Correct Option: (A)**

2. **Solution:** Divide by x: $\frac{dy}{dx}-\frac{1}{x}y=2x$. $P=-1/x$. $IF=e^{\int -1/x dx}=1/x$. **Correct Option: (C)**

3. **Solution:** $ydx+xdy+x^2ydy=0 \implies d(xy)+x^2ydy=0$. Divide by x^2y^2: $\frac{d(xy)}{(xy)^2}+\frac{1}{y}dy=0 \implies -1/xy+\log y=C$. **Correct Option: (A)**

4. **Solution:** The highest order derivative $\frac{d^2y}{dx^2}$ is inside a sine function. Thus, the equation is not a polynomial in derivatives. **Correct Option: (D)**

5. **Solution:** Homogeneous. $y=vx \implies v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2vx^2}=\frac{v^2-1}{2v}$. $x\frac{dv}{dx}=\frac{v^2-1-2v^2}{2v}=-\frac{v^2+1}{2v}$. $\int \frac{2v}{v^2+1}dv=-\int \frac{dx}{x} \implies \log(v^2+1)=-\log x+\log C \implies x(v^2+1)=C$. $x(y^2/x^2+1)=C \implies (y^2+x^2)/x=C \implies x^2+y^2=Cx$. **Correct Option: (A)**

6. **Solution:** $\frac{dy}{y}=\tan x dx \implies \log y=\log\sec x+C$. For $x=0, y=1$, $C=0$. $y=\sec x$. **Correct Option: (B)**

7. **Solution:** $y'=-a\sin(x+b)$, $y''=-a\cos(x+b)=-y$. So $y''+y=0$. **Correct Option: (A)**

8. **Solution:** $IF=e^{\int \frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$. Solution: $y\log x=\int \frac{1}{x}\log x dx=\frac{(\log x)^2}{2}+C$. **Correct Option: (A)**

9. **Solution:** Since it is a homogeneous equation, $y=vx$ is the standard substitution. **Correct Option: (B)**

10. **Solution:** Let $x+y=v \implies 1+dy/dx=dv/dx$. $(v+1)/(v-1)=dv/dx-1 \implies dv/dx=\frac{v+1+v-1}{v-1}=\frac{2v}{v-1}$. $\int \frac{v-1}{v}dv=2\int dx \implies v-\log v=2x+C \implies x+y-\log(x+y)=2x+C \implies y-x-\log(x+y)=C$. **Correct Option: (A)**

11. **Solution:** Standard result for Linear Differential Equations. **Correct Option: (A)**

12. **Solution:** $y=C_1e^x+C_2e^{2x}+C_3e^x\cdot e^{C_4}=(C_1+C_3e^{C_4})e^x+C_2e^{2x}$. Let $A=C_1+C_3e^{C_4}$. Then $y=Ae^x+C_2e^{2x}$. Only 2 independent arbitrary constants. **Correct Option: (C)**

13. **Solution:** $\int \frac{dy}{\sqrt{4-y^2}}=\int dx \implies \sin^{-1}(y/2)=x+C$. **Correct Option: (B)**

14. **Solution:** $\frac{dx}{dy}=\frac{x+2y^3}{y}=\frac{1}{y}x+2y^2$. $\frac{dx}{dy}-\frac{1}{y}x=2y^2$. $IF=1/y$. $x/y=\int 2y^2\cdot\frac{1}{y}dy=y^2+C \implies x=y^3+Cy$. **Correct Option: (A)**

15. **Solution:** $dy/dx=x/y \implies ydy=xdx \implies y^2/2=x^2/2+C \implies y^2-x^2=2C$. This is the equation of a hyperbola. **Correct Option: (C)**

SOLUTIONS

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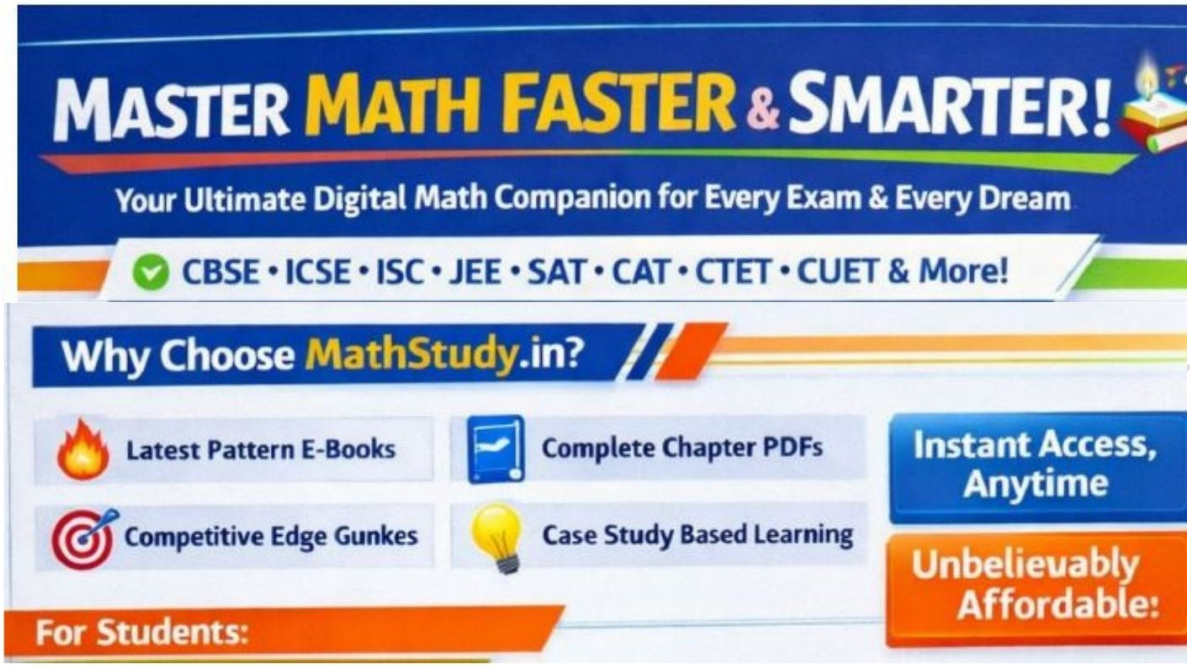

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