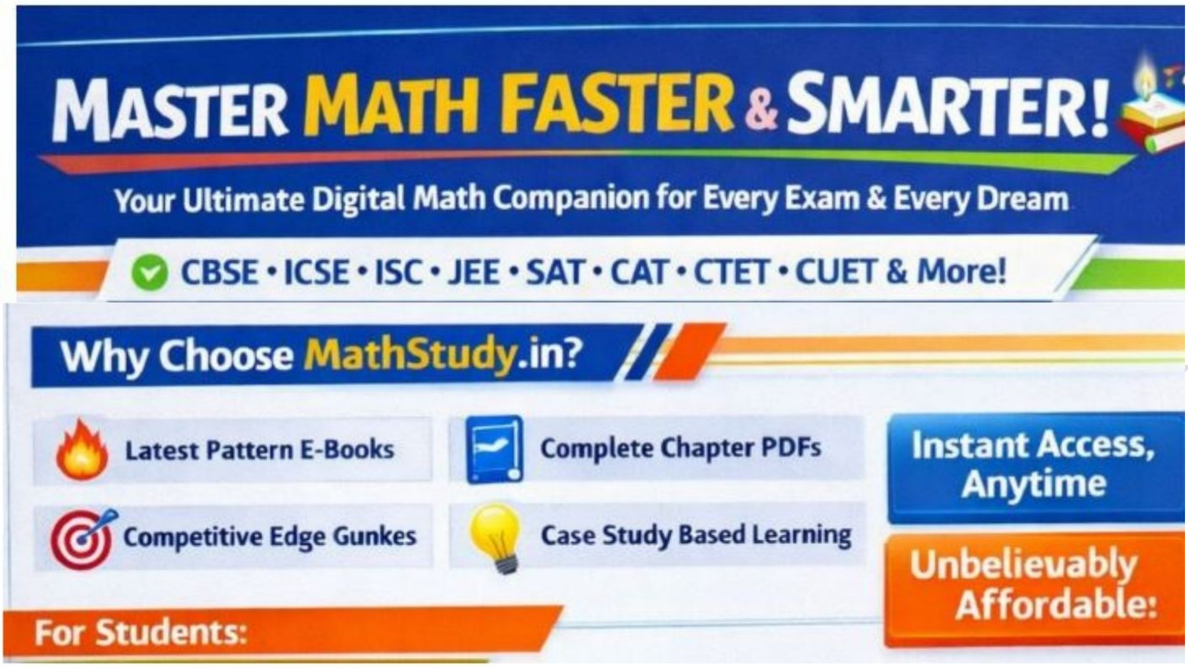
CUET Mathematics Test - Set 16

Chapter: Calculus - Continuity and Differentiability (Intermediate)

SOLUTIONS



Special Features

- **Board-Specific** – CBSE, ICSE, ISC, State Boards
- **Exam-Focused** – JEE, SAT, CAT, CTET, CUET, NTSE
- **Grade-Wise** – Class 6 to 12
- **Bilingual Options** – English & Hindi Medium Support
- **Printable & Shareable** – Use offline, anytime

How to Order:

Visit : https://www.mathstudy.in
Browse by Exam, Class, or Topic
Add to Cart & Checkout

Contact & Support:

📧 Email: admin@mathstudy.in
💬 WhatsApp Support Available : +91-+91 92118 65759

💡 Why Wait? Empower your learning journey, save time, and achieve your dreams!

🌐 Explore & Start Learning Today:
https://www.mathstudy.in – Premium eBooks for success
https://www.udgamwelfarefoundation.com – Free PDFs, practice tests, & guida

MathStudy.in — Empowering Learners, Enabling Educators, Encouraging Excellence.
Digital Learning | Affordable Excellence | Trusted by Thousands

Solutions

1. **Solution:** Continuous as $\lim_{x\to 0} x^2 \sin(1/x) = 0$. $f'(0) = \lim_{h\to 0} \frac{h^2 \sin(1/h)-0}{h} = \lim_{h\to 0} h\sin(1/h) = 0$. **Correct Option: (C)**

2. **Solution:** $y = \sqrt{1+x^2}$. $dy/dx = \frac{x}{\sqrt{1+x^2}}$. At $x = 1$, $dy/dx = 1/\sqrt{2}$. **Correct Option: (A)**

3. **Solution:** $y = \sin^{-1}(e^x)$. $dy/dx = \frac{1}{\sqrt{1-(e^x)^2}} \cdot \frac{d}{dx}(e^x) = \frac{e^x}{\sqrt{1-e^{2x}}}$. **Correct Option: (A)**

4. **Solution:** Take log: $y\log x = x\log y$. Diff: $y'\log x + y/x = 1\cdot\log y + x(1/y)y'$. Collect y' terms: $y'(\log x - x/y) = \log y - y/x$. **Correct Option: (B)**

5. **Solution:** Standard second order relation for $\sin(m\sin^{-1}x)$. $y_1 = \frac{m\cos(m\sin^{-1}x)}{\sqrt{1-x^2}}$. Squaring and differentiating again gives $(1-x^2)y_2 - xy_1 = -m^2 y$. **Correct Option: (B)**

6. **Solution:** $y = \tan^{-1}\left(\frac{\cos^2(x/2)-\sin^2(x/2)}{(\cos(x/2)+\sin(x/2))^2}\right) = \tan^{-1}(\tan(\pi/4 - x/2)) = \pi/4 - x/2$. $y' = -1/2$. **Correct Option: (B)**

7. **Solution:** If $x > 0, f(x) = \log x \Rightarrow 1/x$. If $x < 0, f(x) = \log(-x) \Rightarrow \frac{1}{-x}(-1) = 1/x$. **Correct Option: (B)**

8. **Solution:** $xy = \sqrt{a^{\sin^{-1}t}\cdot a^{\cos^{-1}t}} = \sqrt{a^{\pi/2}}$ (Constant). Diff: $xy' + y(1) = 0 \Rightarrow y' = -y/x$. **Correct Option: (B)**

9. **Solution:** At $x = \pi/4, \cos x > 0$, so $f(x) = \cos x$. $f'(x) = -\sin x$. $f'(\pi/4) = -1/\sqrt{2}$. **Correct Option: (B)**

10. **Solution:** Let $x = \sin A$ and $\sqrt{x} = \sin B$. $y = \sin^{-1}(\sin A\cos B - \cos A\sin B) = A - B = \sin^{-1}x - \sin^{-1}\sqrt{x}$. Diff: $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$. **Correct Option: (A)**

11. **Solution:** At $x = 0, f'(0) = \lim_{h\to 0}\frac{\frac{h}{1+|h|}-0}{h} = \lim_{h\to 0}\frac{1}{1+|h|} = 1$. **Correct Option: (B)**

12. **Solution:** $y_1 = e^{a\cos^{-1}x}\cdot\frac{-a}{\sqrt{1-x^2}} \Rightarrow y_1\sqrt{1-x^2} = -ay$. Square and differentiate: $(1-x^2)y_1^2 = a^2y^2 \Rightarrow (1-x^2)2y_1y_2 - 2xy_1^2 = 2a^2yy_1$. Divide by $2y_1$. **Correct Option: (B)**

13. **Solution:** The factor $|x-3|$ is non-differentiable at $x = 3$. Since $\cos(3) \neq 0$, the product remains non-differentiable there. **Correct Option: (C)**

14. **Solution:** $y = \frac{\log x}{\log a} + \frac{\log a}{\log x} + 1 + 1$. $y' = \frac{1}{\log a}\cdot\frac{1}{x} + \log a\cdot\frac{-1}{(\log x)^2}\cdot\frac{1}{x}$. **Correct Option: (A)**

15. **Solution:** $y = \tan^{-1}a + \tan^{-1}x$. Since $\tan^{-1}a$ is constant, $y' = 0 + \frac{1}{1+x^2}$. **Correct Option: (B)**