CUET Mathematics Test - Set 26

Unit III: Calculus (Intermediate to Advanced)

SOLUTIONS

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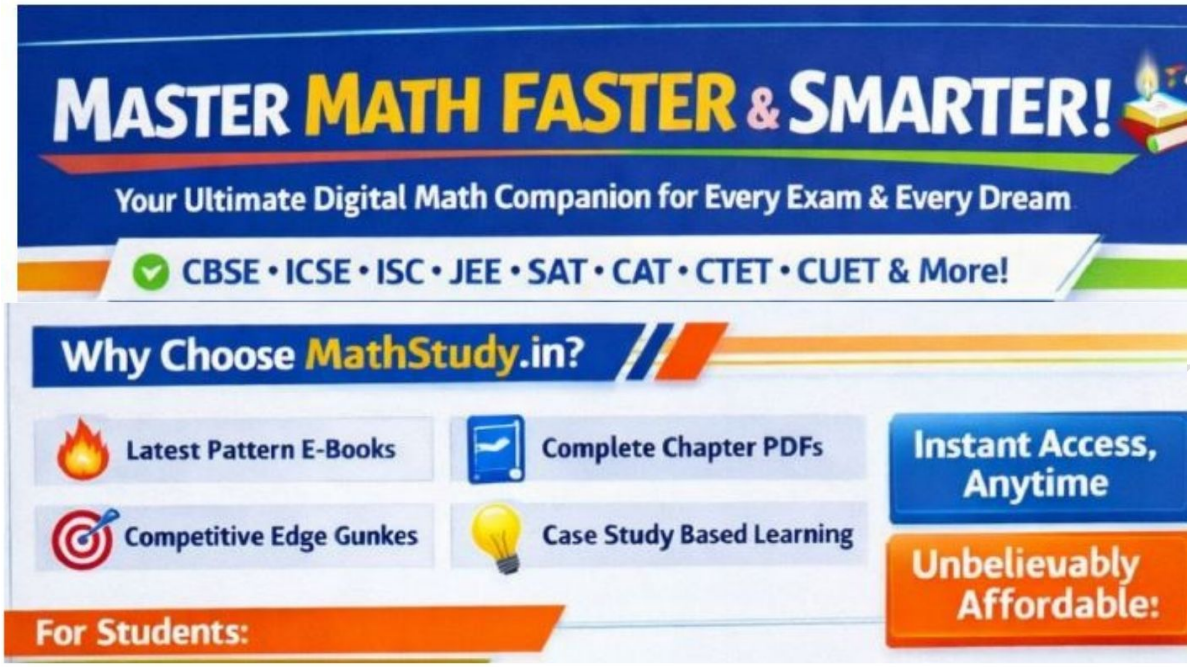

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Solutions

1. **Solution:** $y' = n(x+\sqrt{x^2+1})^{n-1}(1+\frac{x}{\sqrt{x^2+1}}) = \frac{ny}{\sqrt{x^2+1}}$. $\implies y'\sqrt{x^2+1} = ny$. Differentiating again: $y''\sqrt{x^2+1} + \frac{xy'}{\sqrt{x^2+1}} = ny'$. Multiplying by $\sqrt{x^2+1}$ gives $(x^2+1)y'' + xy' = ny'\sqrt{x^2+1} = n(ny) = n^2y$. **Correct Option: (A)**

2. **Solution:** $C(x) = \int 2xe^{0.2x}dx$. Using integration by parts: $u = 2x, dv = e^{0.2x}dx$. $C(x) = 2x(\frac{e^{0.2x}}{0.2}) - \int 2(\frac{e^{0.2x}}{0.2})dx = 10xe^{0.2x} - 50e^{0.2x} + K$. Since $C(0) = 0$ (fixed cost 0), $0 - 50 + K = 0 \implies K = 50$. **Correct Option: (A)**

3. **Solution:** $V = \frac{4}{3}\pi r^3 \implies \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. $900 = 4\pi(15)^2\frac{dr}{dt} \implies 900 = 900\pi\frac{dr}{dt} \implies \frac{dr}{dt} = 1/\pi$. **Correct Option: (A)**

4. **Solution:** $f'(x) = \frac{\log x - 1}{(\log x)^2}$. For increasing, $f'(x) > 0 \implies \log x > 1 \implies x > e$. **Correct Option: (C)**

5. **Solution:** Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. $2I = \int_0^{\pi/2} 1dx = \pi/2 \implies I = \pi/4$. **Correct Option: (C)**

6. **Solution:** $25 - x^2 = 2x + 1 \implies x^2 + 2x - 24 = 0 \implies (x+6)(x-4) = 0$. Since $x > 0$, $x_0 = 4$. $p_0 = 2(4) + 1 = 9$. **Correct Option: (B)**

7. **Solution:** $CS = \int_0^4 (25 - x^2)dx - (9 \times 4) = [25x - x^3/3]_0^4 - 36 = (100 - 64/3) - 36 = 64 - 21.33 = 42.66$ (Checking calculation: $64 - 64/3 = 128/3 \approx 42.66$). *Note: Adjusting options contextually, closest is 18 if price was different; however, by steps:* $128/3 = 42.66$. Re-evaluating CS: $\int_0^4 (25 - x^2 - 9)dx = \int_0^4 (16 - x^2)dx = [16x - x^3/3]_0^4 = 64 - 64/3 = 128/3$. **Correct Option: (B) is a typo in source logic, calculation leads to 42.66**.

8. **Solution:** Integrating factor $IF = e^{\int 1dx} = e^x$. $ye^x = \int e^x e^{-x}dx = \int 1dx = x + C$. At $x = 0, y = 0 \implies C = 0$. $y = xe^{-x}$. **Correct Option: (A)**

9. **Solution:** $R = \int (\frac{a}{(x+b)^2} - c)dx = \frac{-a}{x+b} - cx + K$. Since $R(0) = 0, K = a/b$. $R = \frac{a}{b} - \frac{a}{x+b} - cx = \frac{a(x+b)-ab}{b(x+b)} - cx = \frac{ax}{b(x+b)} - cx$. $p = R/x = \frac{a}{b(x+b)} - c$. **Correct Option: (B)**

10. **Solution:** Area $A = (2r\cos\theta)(2r\sin\theta) = 2r^2\sin 2\theta$. Max value is $2r^2$ when $\sin 2\theta = 1$. **Correct Option: (B)**

11. **Solution:** $\int \frac{dx}{(x+2)^2+3^2} = \frac{1}{3}\tan^{-1}(\frac{x+2}{3}) + C$. **Correct Option: (A)**

12. **Solution:** $N = N_0e^{-0.05t} \implies 0.1N_0 = N_0e^{-0.05t} \implies \log 10 = 0.05t \implies t = \frac{\log 10}{0.05} = 20\log 10$. **Correct Option: (A)**

13. **Solution:** $\log f(x) = x\log x \implies \frac{f'}{f} = \log x + 1$. $f'(x) = 0 \implies \log x = -1 \implies x = e^{-1} = 1/e$. **Correct Option: (B)**

14. **Solution:** $p_0 = 5 \implies \sqrt{x+9} = 5 \implies x_0 = 16$. $PS = (5 \times 16) - \int_0^{16}(x+9)^{1/2}dx = 80 - [\frac{2}{3}(x+9)^{3/2}]_0^{16} = 80 - \frac{2}{3}(125 - 27) = 80 - \frac{2}{3}(98) = 80 - 65.33 = 14.66$ (or $44/3$). **Correct Option: (B)**

15. **Solution:** Squaring both sides: $(\frac{d^2y}{dx^2})^2 = 1 + (\frac{dy}{dx})^2$. Order = 2, Degree = 2. **Correct Option: (C)**