Chapter 6: Triangles

This chapter explores when two triangles are identical in shape and size (congruent), and the fundamental relationships between the sides and angles within a single triangle.

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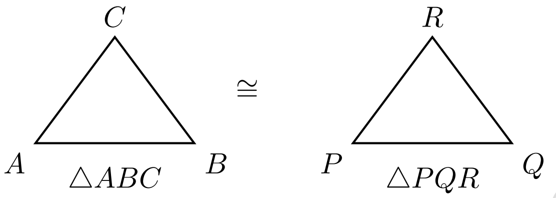
1. Congruence of Triangles

Two triangles are congruent if all corresponding sides and angles are equal. The following criteria help us prove congruence without checking all six elements.

1. **Side-Side-Side (SSS) Congruence Rule**

   - **Condition**: If three sides of one triangle are equal to three sides of another triangle.
   - **Usage**: When all side lengths are known or can be proved equal.

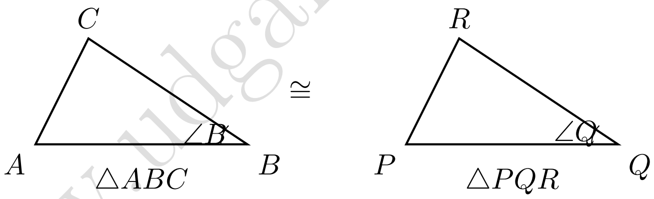
$$\text{If } AB = PQ,\ BC = QR,\ CA = RP \text{ then } \triangle ABC \cong \triangle PQR$$



2. **Side-Angle-Side (SAS) Congruence Rule**

   - **Condition**: If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle.
   - **Usage**: When two sides and the angle between them are known to be equal.

$$\text{If } AB = PQ,\ \angle B = \angle Q,\ BC = QR \text{ then } \triangle ABC \cong \triangle PQR$$



3. **Angle-Side-Angle (ASA) Congruence Rule**

   - **Condition**: If two angles and the included side of one triangle are equal to two angles and the included side of another triangle.
   - **Usage**: When two angles and the side between them are known to be equal.

$$\text{If } \angle B = \angle Q,\ BC = QR,\ \angle C = \angle R \text{ then } \triangle ABC \cong \triangle PQR$$

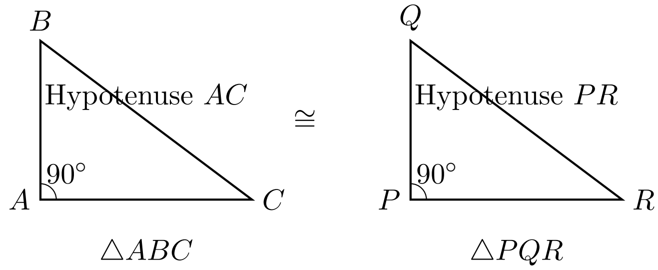
4. **Angle-Angle-Side (AAS) Congruence Rule**

   - **Condition**: If two angles and a non-included side of one triangle are equal to the corresponding two angles and side of another triangle.
   - **Usage**: A variation of ASA; very useful in proofs.

$$\text{If } \angle A = \angle P,\ \angle B = \angle Q,\ BC = QR \text{ then } \triangle ABC \cong \triangle PQR$$

5. **Right Angle-Hypotenuse-Side (RHS) Congruence Rule**
   - **Condition**: If the hypotenuse and one side of a right triangle are equal to the hypotenuse and one side of another right triangle.
   - **Usage**: Only for right-angled triangles. Very important specific case.

$$\text{If } \angle B = \angle Q = 90^\circ,\ AC = PR,\ BC = QR \text{ then } \triangle ABC \cong \triangle PQR$$



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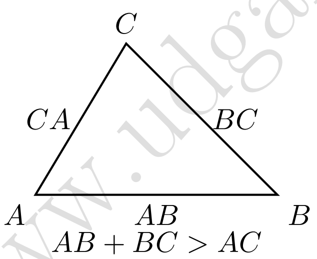
2. Properties of Triangle Inequalities

These theorems describe the relationships between sides and angles within a single triangle.

6. **Triangle Inequality Theorem**

$$AB + BC > AC, \quad BC + CA > AB, \quad CA + AB > BC$$

   - AB, BC, CA: Sides of triangle ABC.
   - **Meaning**: The sum of any two sides of a triangle is greater than the third side.
   - **Usage**: To check if three given lengths can form a triangle.



7. **Angle Opposite to Longer Side**

$$\text{If } AB > AC \text{ then } \angle C > \angle B$$

   - **Meaning**: In any triangle, the angle opposite the longer side is larger.
   - **Usage**: To compare angles when side lengths are known.

8. **Side Opposite to Larger Angle**

$$\text{If } \angle C > \angle B \text{ then } AB > AC$$

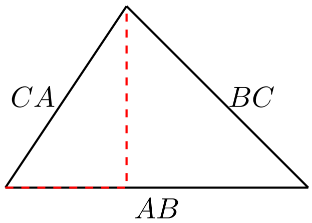
   - **Meaning**: In any triangle, the side opposite the larger angle is longer.

- **Usage**: To compare side lengths when angles are known. This is the converse of the previous property.

9. **Sum of Any Two Sides**

$$AB + AC > BC$$

- **Usage**: A specific application of the triangle inequality. The shortest distance between two points (B and C) is the straight line BC.



$AB + AC > BC$

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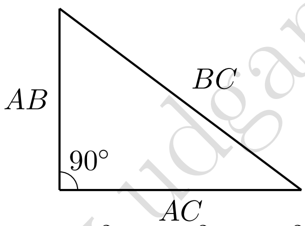
3. Important Results about Triangles

10. **Pythagoras Theorem (For Right Triangles Only)**

$$AC^2 = AB^2 + BC^2$$

- **Condition**: Only applies when $\angle B = 90°$.
- AC: Hypotenuse (side opposite the right angle).
- AB, BC: The other two sides (legs).
- **Usage**: To find the length of any side when the other two are known in a right triangle.



$AC^2 = AB^2 + BC^2$

11. **Exterior Angle Inequality**

$$\angle ACD > \angle A \quad \text{and} \quad \angle ACD > \angle B$$

- $\angle ACD$: An exterior angle of $\triangle ABC$.
- **Meaning**: An exterior angle is greater than either of the opposite interior angles.
- **Usage**: Used in inequality proofs involving triangles.

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Quick Revision Summary

Here are all the essential formulas and criteria from this chapter.

1. **SSS Congruence**: If $AB = PQ,\ BC = QR,\ CA = RP$ then $\triangle ABC \cong \triangle PQR$
2. **SAS Congruence**: If $AB = PQ,\ \angle B = \angle Q,\ BC = QR$ then $\triangle ABC \cong \triangle PQR$
3. **ASA Congruence**: If $\angle B = \angle Q,\ BC = QR,\ \angle C = \angle R$ then $\triangle ABC \cong \triangle PQR$
4. **AAS Congruence**: If $\angle A = \angle P,\ \angle B = \angle Q,\ BC = QR$ then $\triangle ABC \cong \triangle PQR$
5. **RHS Congruence**: If $\angle B = \angle Q = 90°,\ AC = PR,\ BC = QR$ then $\triangle ABC \cong \triangle PQR$
6. **Triangle Inequality**: $AB + BC > AC$, $BC + CA > AB$, $CA + AB > BC$
7. **Angle Opposite Longer Side**: If $AB > AC$ then $\angle C > \angle B$
8. **Side Opposite Larger Angle**: If $\angle C > \angle B$ then $AB > AC$
9. **Pythagoras Theorem**: $AC^2 = AB^2 + BC^2$ (when $\angle B = 90°$)
10. **Exterior Angle Inequality**: $\angle ACD > \angle A$ and $\angle ACD > \angle B$