CUET (UG) – MATHEMATICS

Chapter Test - Unit IV: Calculus - Vectors and Three-Dimensional Geometry

SOLUTIONS

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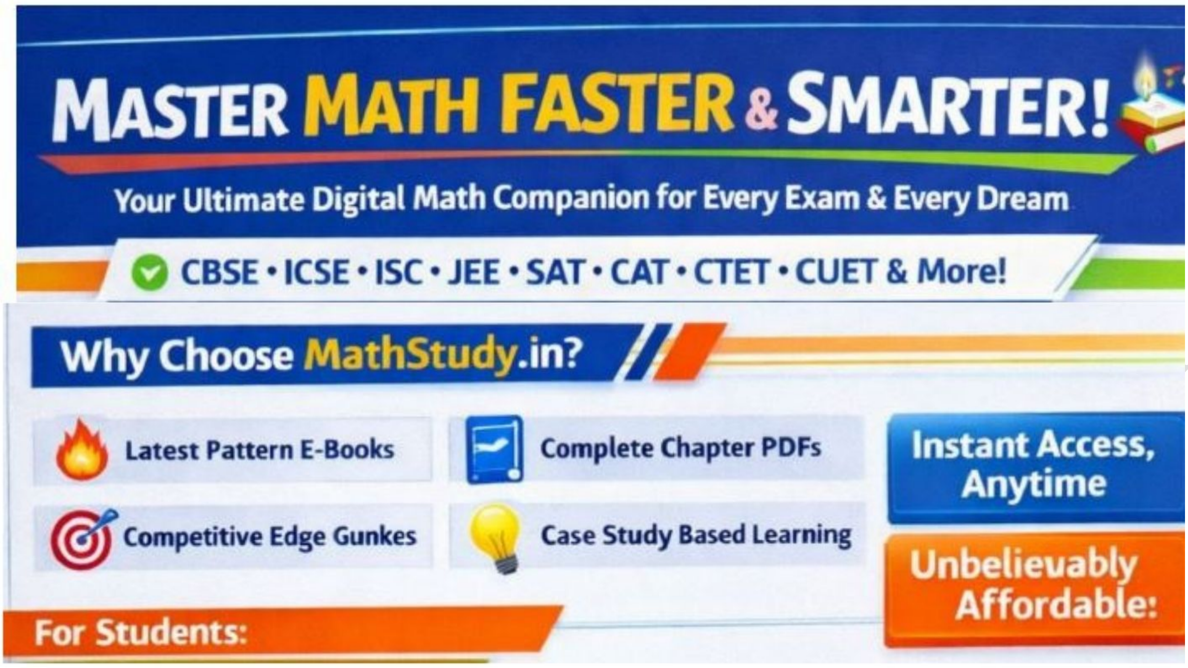

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Solutions

1. **Correct Option: (B)**. Let the angle be θ. $2\vec{a}$ is in the same direction as $\vec{a}$, but $-3\vec{b}$ is in the opposite direction to $\vec{b}$. Thus, the angle is $\pi - \pi/6 = 5\pi/6$.

2. **Correct Option: (C)**. $\hat{j} \times \hat{k} = \hat{\imath}$, $\hat{\imath} \times \hat{k} = -\hat{j}$, $\hat{\imath} \times \hat{j} = \hat{k}$. Result: $\hat{\imath} \cdot \hat{\imath} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k} = 1 - 1 + 1 = 1$.

3. **Correct Option: (B)**. The z-axis is the line $x = 0, y = 0$. By calculating the point on the given line closest to the origin in the xy-projection, the distance is found to be 2.

4. **Correct Option: (C)**. $\cos^2 60^\circ + \cos^2 45^\circ + \cos^2\gamma = 1 \implies 1/4 + 1/2 + \cos^2\gamma = 1 \implies \cos^2\gamma = 1/4 \implies \gamma = 60^\circ$.

5. **Correct Option: (A)**. Let $A(1, 1, 2), B(2, 3, 5), C(1, 5, 5)$. $\vec{AB} = (1, 2, 3)$, $\vec{AC} = (0, 4, 3)$. $\vec{AB} \times \vec{AC} = (-6, -3, 4)$. Area $= \frac{1}{2}\sqrt{36 + 9 + 16} = \sqrt{61}/2$.

6. **Correct Option: (D)**. Projection on y-axis is the y-component of the vector. Here $\vec{b} = 3\hat{\imath} + 0\hat{j} + 4\hat{k}$, so the magnitude is 0.

7. **Correct Option: (D)**. $\cos\theta = -1$, which occurs at $\theta = 180^\circ$.

8. **Correct Option: (C)**. The distance from the xy-plane ($z = 0$) is simply the absolute value of the z-coordinate, $|c|$.

9. **Correct Option: (D)**. $DRs = (-1 - 2, 3 - 3, 2 - 5) = (-3, 0, -3)$. Any multiple is also a DR, so $(3, 0, 3)$ is also correct.

10. **Correct Option: (A)**. Parallel lines share the same direction ratios $(2, -3, 8)$. The line passes through $(1, 2, 3)$.

11. **Correct Option: (B)**. $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = (100)(4) - 144 = 256$. Thus, $|\vec{a} \times \vec{b}| = 16$.

12. **Correct Option: (A)**. Rewrite in standard form: $\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ and $\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$. Condition: $(-3)(-3p/7) + (2p/7)(1) + (2)(-5) = 0 \implies 9p/7 + 2p/7 = 10 \implies 11p = 70$.

13. **Correct Option: (B)**. $|\vec{a} + \vec{b} + \vec{c}|^2 = \sum |\vec{a}|^2 + 2(\sum \vec{a} \cdot \vec{b}) = 0$. Since they are unit vectors, $3 + 2(\sum \vec{a} \cdot \vec{b}) = 0 \implies \sum \vec{a} \cdot \vec{b} = -3/2$.

14. **Correct Option: (A)**. The line is parallel to $(3, -1, 2)$. The x-axis is parallel to $(1, 0, 0)$. $\cos\theta = \frac{3(1)+0+0}{\sqrt{9+1+4}\sqrt{1}} = 3/\sqrt{14}$.

15. **Correct Option: (B)**. Cross product $(\hat{\imath} + \hat{j}) \times (\hat{j} + \hat{k}) = \hat{\imath} - \hat{j} + \hat{k}$. Unit vector is obtained by dividing by $\sqrt{3}$.

16. **Correct Option: (A)**. $\cos^2(\pi/4) + \cos^2(\pi/4) + \cos^2\gamma = 1 \implies 1/2 + 1/2 + \cos^2\gamma = 1 \implies \cos\gamma = 0 \implies \gamma = \pi/2$.

17. **Correct Option: (A)**. Midpoint $= (\frac{2+4}{2}, \frac{3+1}{2}, \frac{4-2}{2}) = (3, 2, 1)$.

18. **Correct Option: (D)**. Distance $= \sqrt{(1-2)^2 + (-1+3)^2 + (3-1)^2} = \sqrt{1 + 4 + 4} = 3$.

19. **Correct Option: (C)**. $\cos\theta = \frac{1(\sqrt{3}-1)+1(-\sqrt{3}-1)+2(4)}{\sqrt{1+1+4}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+16}} = \frac{-2+8}{\sqrt{6}\sqrt{4-2\sqrt{3}+4+2\sqrt{3}+16}} = \frac{6}{\sqrt{6}\sqrt{24}} = \frac{6}{\sqrt{144}} = 6/12 = 1/2$. $\theta = 60^\circ$.

20. **Correct Option: (B)**. The magnitude of $\frac{1}{2}(\vec{a} \times \vec{b})$ gives the area, and the vector itself is the vector area.