

Chapter 8: Quadrilaterals

This chapter explores the properties of four-sided figures called quadrilaterals. You will learn about special types of quadrilaterals and their unique characteristics, as well as an important theorem about triangles.

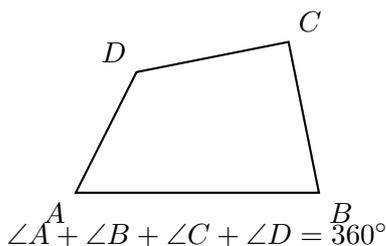
1. Basic Properties of Quadrilaterals

Every quadrilateral shares these fundamental properties.

1. Angle Sum Property of a Quadrilateral

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

- $\angle A, \angle B, \angle C, \angle D$: The four interior angles of any quadrilateral.
- **Meaning**: The sum of all interior angles of a quadrilateral is always 360° .
- **Usage**: To find the fourth angle when three angles are known, or to verify if four given angles can form a quadrilateral.



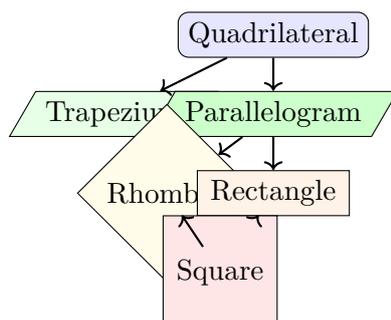
2. Sum of All Exterior Angles of a Quadrilateral

$$\text{Sum of all exterior angles} = 360^\circ$$

- **Meaning**: For any polygon (including quadrilateral), the sum of exterior angles, one at each vertex, is always 360° , regardless of the number of sides.
- **Usage**: Useful in problems involving exterior angles.

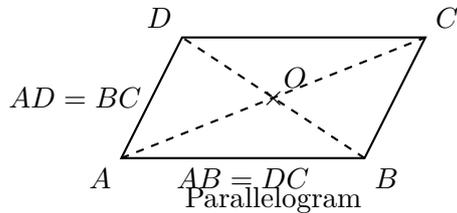
2. Special Quadrilaterals and Their Properties

Different types of quadrilaterals have additional special properties. Remember: each type inherits all properties of the types above it in the hierarchy.



3. Properties of a Parallelogram

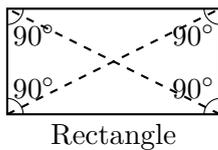
- Opposite sides are parallel and equal: $AB \parallel DC$, $AB = DC$ and $AD \parallel BC$, $AD = BC$
- Opposite angles are equal: $\angle A = \angle C$ and $\angle B = \angle D$
- Diagonals bisect each other: $AO = OC$ and $BO = OD$
- Adjacent angles are supplementary: $\angle A + \angle B = 180^\circ$



4. Properties of a Rectangle (A type of parallelogram)

$$AC = BD \quad (\text{Diagonals are equal})$$

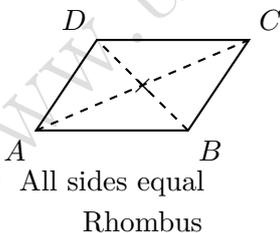
- All properties of parallelogram apply.
- Each interior angle = 90°
- Diagonals are equal in length.



5. Properties of a Rhombus (A type of parallelogram)

$$AC \perp BD \quad (\text{Diagonals are perpendicular})$$

- All properties of parallelogram apply.
- All sides are equal: $AB = BC = CD = DA$
- Diagonals are perpendicular to each other.
- Diagonals bisect the opposite angles.



6. Properties of a Square (A type of rectangle and rhombus)

$$AC = BD \quad \text{and} \quad AC \perp BD$$

- All properties of rectangle and rhombus apply.
- All sides equal, all angles 90° .
- Diagonals equal and perpendicular.

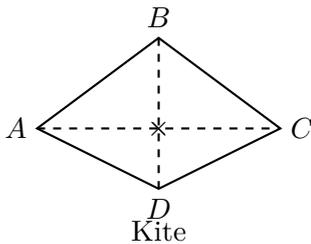


All sides equal
Square

7. Properties of a Kite

$$AC \perp BD$$

- Two pairs of adjacent sides equal: $AB = AD$ and $CB = CD$
- One diagonal bisects the other at 90° .
- One pair of opposite angles are equal.



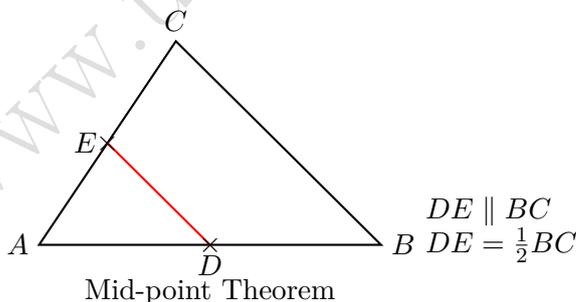
3. Mid-point Theorem

This important theorem connects the midpoints of sides of a triangle.

9. Mid-point Theorem Statement

If D and E are midpoints of AB and AC respectively, then $DE \parallel BC$ and $DE = \frac{1}{2}BC$

- D, E : Midpoints of sides AB and AC of $\triangle ABC$.
- DE : Line segment joining the midpoints.
- **Meaning:** The line segment joining midpoints of two sides is parallel to the third side and half its length.
- **Usage:** To prove parallelism or find lengths in triangle problems.



10. Converse of Mid-point Theorem

If D is midpoint of AB and $DE \parallel BC$ meeting AC at E , then E is midpoint of AC

- **Usage:** To prove a point is a midpoint when parallelism is known.

4. Conditions for Special Quadrilaterals

These help prove what type of quadrilateral a given figure is.

11. Condition for a Parallelogram

- If both pairs of opposite sides are parallel.
- If both pairs of opposite sides are equal.
- If diagonals bisect each other.
- If a pair of opposite sides are both parallel and equal.

12. Condition for a Rectangle

- If it is a parallelogram with one angle 90° .
- If diagonals are equal and bisect each other.

13. Condition for a Rhombus

- If it is a parallelogram with adjacent sides equal.
- If diagonals are perpendicular and bisect each other.

14. Condition for a Square

- If it is a rectangle with adjacent sides equal.
- If it is a rhombus with one angle 90° .

Quick Revision Summary

Here are all the essential formulas and properties from this chapter.

1. **Angle Sum Property:** $\angle A + \angle B + \angle C + \angle D = 360^\circ$

2. **Parallelogram Properties:**

- Opposite sides \parallel and equal
- Opposite angles equal
- Diagonals bisect each other

3. **Rectangle Properties:** All parallelogram properties + Each angle 90° + Diagonals equal

4. **Rhombus Properties:** All parallelogram properties + All sides equal + Diagonals \perp

5. **Square Properties:** All rectangle and rhombus properties

6. **Kite Properties:** Two pairs adjacent sides equal + One diagonal \perp bisector of other

7. **Mid-point Theorem:** $DE \parallel BC$ and $DE = \frac{1}{2}BC$

8. **Converse of Mid-point Theorem:** If D is midpoint of AB and $DE \parallel BC$, then E is midpoint of AC