

# Chapter 15: Probability

This chapter introduces the mathematics of chance. You'll learn how to measure how likely an event is to occur, from impossible (0) to certain (1).

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## 1. Basic Definitions

### 1. Experiment

- An action that leads to well-defined outcomes.
- **Examples:** Tossing a coin, rolling a die, drawing a card.

### 2. Random Experiment

- An experiment where all possible outcomes are known, but the exact result cannot be predicted in advance.

### 3. Sample Space ( $S$ )

$S = \{\text{Set of all possible outcomes}\}$

- **Examples:**
  - Coin toss:  $S = \{\text{Head, Tail}\}$
  - Die roll:  $S = \{1, 2, 3, 4, 5, 6\}$

### 4. Event ( $E$ )

$E \subseteq S$  (Event is a subset of sample space)

- A collection of one or more outcomes.
- **Examples:**
  - Getting an even number on die:  $E = \{2, 4, 6\}$
  - Getting a Head on coin:  $E = \{\text{Head}\}$

### 5. Favorable Outcomes

Outcomes that satisfy the event condition

- **Example:** For event "rolling a number  $> 4$ " on die, favorable outcomes =  $\{5, 6\}$ .
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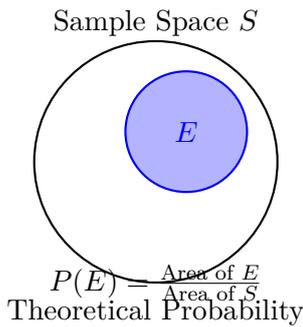
## 2. Types of Probability

### 6. Theoretical (Classical) Probability

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{n(E)}{n(S)}$$

- $P(E)$ : Probability of event  $E$  occurring.
- $n(E)$ : Number of favorable outcomes for  $E$ .
- $n(S)$ : Total number of equally likely outcomes in sample space.
- **Condition:** All outcomes must be equally likely.

- **Range:**  $0 \leq P(E) \leq 1$



### 7. Experimental (Empirical) Probability

$$P(E) = \frac{\text{Number of trials in which event occurred}}{\text{Total number of trials conducted}}$$

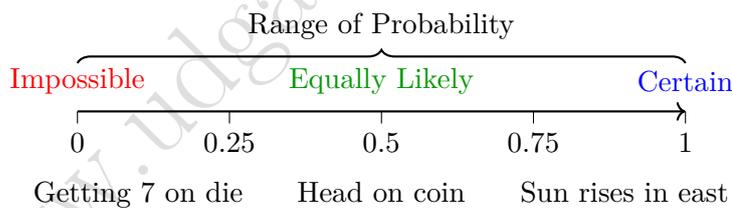
- Also called **relative frequency**.
- Based on actual experiments/observations.
- As number of trials increases, experimental probability approaches theoretical probability.

## 3. Important Properties and Rules

### 8. Range of Probability

$$0 \leq P(E) \leq 1$$

- $P(E) = 0$ : Impossible event (e.g., getting 7 on a standard die).
- $P(E) = 1$ : Certain event (e.g., getting a number between 1-6 on a die).
- $0 < P(E) < 1$ : Event is possible but not certain.



### 9. Complementary Events

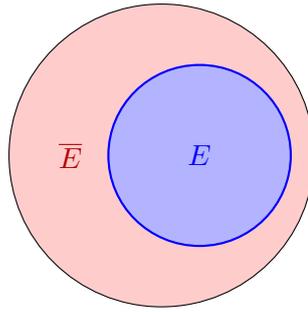
$$P(E) + P(\text{not } E) = 1$$

$$P(\text{not } E) = 1 - P(E)$$

An event that represents the non-occurrence of event  $E$  is called its complement.

- not  $E$  or  $\bar{E}$ :** This represents the event that  $E$  does not occur.
- Usage:** It provides an easy way to find the probability of an event not happening when the total probability is known.

$$P(E) + P(\bar{E}) = 1$$



$$P(E) + P(\bar{E}) = 1$$

### Quick Revision Summary

- (a)  $P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$
- (b)  $0 \leq P(E) \leq 1$
- (c)  $P(\bar{E}) = 1 - P(E)$

### 10. Sum of Probabilities of All Elementary Events

$$P(E_1) + P(E_2) + \dots + P(E_n) = 1$$

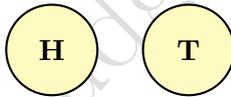
- $E_1, E_2, \dots, E_n$ : All possible elementary (simple) events in sample space.
- **Example:** For fair die,  $P(1) + P(2) + \dots + P(6) = 1$ .

## 4. Common Probability Examples

### 11. Coin Toss (Fair Coin)

$$P(\text{Head}) = P(\text{Tail}) = \frac{1}{2}$$

Equally likely



$$S = \{\text{H}, \text{T}\}, P(\text{H}) = P(\text{T}) = \frac{1}{2}$$

### 12. Single Die Roll (Fair Die)

$$P(\text{Any number}) = \frac{1}{6}$$



$$S = \{1, 2, 3, 4, 5, 6\}, P(\text{any number}) = \frac{1}{6}$$

### 13. Deck of Cards (52 Cards)

$$P(\text{Any specific card}) = \frac{1}{52}$$

- **Total cards:** 52

- **Suits:** 13 cards each in deck
- **Face cards:** Jack, Queen, King (12 total)
- **Aces:** 4

Event	Probability
Drawing a Spade	$\frac{13}{52} = \frac{1}{4}$
Drawing a King	$\frac{4}{52} = \frac{1}{13}$
Drawing a Red Card	$\frac{26}{52} = \frac{1}{2}$
Drawing a Face Card	$\frac{12}{52} = \frac{3}{13}$

## 5. Solving Probability Problems

### 14. Step-by-Step Method

- Identify the experiment.
- List all possible outcomes (Sample Space  $S$ ).
- Identify favorable outcomes for the event  $E$ .
- Count:  $n(E)$  and  $n(S)$ .
- Apply:  $P(E) = \frac{n(E)}{n(S)}$ .
- Simplify the fraction if possible.

## 6. Special Cases and Examples

### 15. Sure Event

$$P(\text{Sure event}) = 1$$

- **Example:** Probability that a die roll gives number  $\leq 6$ .

### 16. Impossible Event

$$P(\text{Impossible event}) = 0$$

- **Example:** Probability that a die roll gives number = 7.

### 17. Equally Likely Events

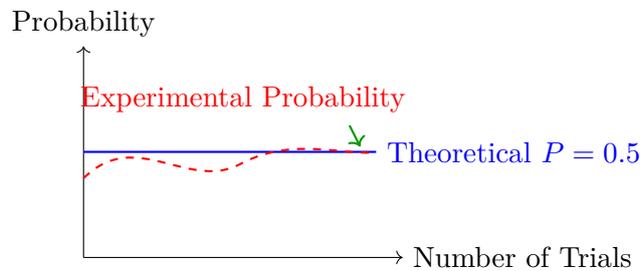
$$P(E_1) = P(E_2) = \dots = P(E_n)$$

- All outcomes have same chance.
- **Example:** Fair coin, fair die.

## 7. Experimental vs Theoretical Probability

### 18. Comparison

- **Theoretical:** Calculated using logic/mathematics.
- **Experimental:** Based on actual trials/observations.
- As number of trials  $\rightarrow \infty$ , experimental probability  $\rightarrow$  theoretical probability.



Experimental approaches Theoretical as trials increase

## 8. Real-Life Applications

### 19. Common Applications

- Weather forecasting: "30% chance of rain"
- Games: Card games, dice games, lotteries
- Quality control: Defective items in production
- Insurance: Risk calculation
- Sports: Winning chances

## Quick Revision Summary

Here are all the essential formulas from this chapter.

1. **Theoretical Probability:**  $P(E) = \frac{n(E)}{n(S)} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$
2. **Experimental Probability:**  $P(E) = \frac{\text{Trials with event}}{\text{Total trials}}$
3. **Range of Probability:**  $0 \leq P(E) \leq 1$
4. **Impossible Event:**  $P(E) = 0$
5. **Certain Event:**  $P(E) = 1$
6. **Complementary Events:**  $P(E) + P(\bar{E}) = 1$  or  $P(\bar{E}) = 1 - P(E)$
7. **Sum of Elementary Events:**  $P(E_1) + P(E_2) + \dots + P(E_n) = 1$
8. **Fair Coin:**  $P(H) = P(T) = \frac{1}{2}$

9. **Fair Die:**  $P(\text{any number}) = \frac{1}{6}$

10. **Deck of Cards:**  $P(\text{specific card}) = \frac{1}{52}$

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