SOLUTIONS

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Section A: Multiple Choice Questions

1. **(d) 50.** By Remainder Theorem, $R = p(-1)$. $p(-1) = (-1)^{51} + 51 = -1 + 51 = 50$.
2. **(c)** $6xy$**.** We know if $A + B + C = 0$, then $A^3 + B^3 + C^3 = 3ABC$. Here $x + y + 2 = 0$, so $x^3 + y^3 + 2^3 = 3(x)(y)(2) \implies x^3 + y^3 + 8 = 6xy$.
3. **(a) 8.** $p(x) = x + 4$ and $p(-x) = -x + 4$. Sum $= (x + 4) + (-x + 4) = 8$.
4. **(b) -6.** $(x - 2)^3 = x^3 - 3(x^2)(2) + 3(x)(2^2) - 2^3 = x^3 - 6x^2 + 12x - 8$. Coefficient is -6.
5. **(d) 3.** $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3+b^3+c^3}{abc}$. Since $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$. Thus, $\frac{3abc}{abc} = 3$.
6. **(b)** $x - 1$**.** $(x - 1) - (x^2 - 1) = (x - 1) - (x - 1)(x + 1) = (x - 1)[1 - (x + 1)] = (x - 1)(-x)$.
7. **(c)** n**.** A polynomial of degree n has exactly n complex zeroes, and at most n real zeroes.
8. **(c) 0.** Let $A = x - a, B = x - b, C = x - c$. $A + B + C = 3x - (a + b + c)$. Since $3x = a + b + c$, $A + B + C = 0$. The identity $A^3 + B^3 + C^3 - 3ABC = 0$ holds.

Section B: Very Short Answer Questions

9. $x^2 - 1 = (x - 1)(x + 1)$. Thus $p(1) = 0$ and $p(-1) = 0$.
   $p(1) = a + b + c + d + e = 0$
   $p(-1) = a - b + c - d + e = 0 \implies a + c + e = b + d$.
   Substituting $b + d$ into the first: $(b + d) + (b + d) = 0 \implies 2(b + d) = 0 \implies b + d = 0$.
   Hence $a + c + e = 0$ and $b + d = 0$.
10. Let $x = 28, y = -15, z = -13$. $x + y + z = 28 - 15 - 13 = 0$.
    Value $= 3xyz = 3(28)(-15)(-13) = 16380$.
11. $x^6 - y^6 = (x^3)^2 - (y^3)^2 = (x^3 - y^3)(x^3 + y^3)$
    $= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$.
12. $p(1) = 4(1)^3 + 3(1)^2 - 4(1) + k = 0 \implies 4 + 3 - 4 + k = 0 \implies k = -3$.

Section C: Short Answer Questions

13. $(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2 = 27 - 2 = 25$.
    $x - \frac{1}{x} = \sqrt{25} = \pm 5$.
14. $p(x) = x^3 - 6x^2 + 11x - 6$. $p(1) = 1 - 6 + 11 - 6 = 0$. Factor is $(x - 1)$.
    Dividing: $x^3 - x^2 - 5x^2 + 5x + 6x - 6 = x^2(x - 1) - 5x(x - 1) + 6(x - 1)$.
    $= (x - 1)(x^2 - 5x + 6) = (x - 1)(x - 2)(x - 3)$.
15. Let $X = a + b, Y = b + c, Z = c + a$. Then $X + Y + Z = 2(a + b + c)$.
    Using $X^3 + Y^3 + Z^3 - 3XYZ = \frac{1}{2}(X + Y + Z)[(X - Y)^2 + (Y - Z)^2 + (Z - X)^2]$.
    $(X - Y) = a - c, (Y - Z) = b - a, (Z - X) = c - b$.
    LHS $= \frac{1}{2}(2(a + b + c))[(a - c)^2 + (b - a)^2 + (c - b)^2]$
    $= (a + b + c)[(a - c)^2 + (b - a)^2 + (c - b)^2] = 2(a^3 + b^3 + c^3 - 3abc)$.

Section D: Long Answer Questions

16. (i) $a^3 = 64 \implies a = 4$. Total side $S = a + b = 4 + 2 = 6$.
    Total Volume $= S^3 = 6^3 = 216$ cubic units.
    (ii) Rectangular block dimensions: $a \times a \times b = 4 \times 4 \times 2$.
    Surface Area $= 2(lw + wh + hl) = 2(4 \times 4 + 4 \times 2 + 2 \times 4)$
    $= 2(16 + 8 + 8) = 2(32) = 64$ sq units.

17. We know $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - (xy + yz + zx))$.
    First, find $x^2 + y^2 + z^2$:
    $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
    $1^2 = x^2 + y^2 + z^2 + 2(-1) \implies 1 = x^2 + y^2 + z^2 - 2 \implies x^2 + y^2 + z^2 = 3$.
    Substitute into identity:
    $x^3 + y^3 + z^3 - 3(-1) = (1)(3 - (-1))$
    $x^3 + y^3 + z^3 + 3 = 1(4) \implies x^3 + y^3 + z^3 = 4 - 3 = 1$.