

1 Polynomials

1.1 Definition of Polynomial in One Variable

1. A polynomial in one variable x is an algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where a_n, a_{n-1}, \dots, a_0 are real numbers and n is a non-negative integer.

Here, a_n is the leading coefficient and a_0 is the constant term. Polynomials are used to represent algebraic relationships in equations and graphs.

1.2 Coefficients, Terms and Degree

1. In the polynomial

$$p(x) = 3x^2 - 5x + 7$$

the coefficients are 3, -5 , and 7.

2. Each part such as $3x^2$, $-5x$, and 7 is called a term.
3. The highest power of the variable is called the degree of the polynomial.

Degree helps in classifying polynomials and predicting the shape of their graphs.

1.3 Types of Polynomials

1. Zero polynomial

$$p(x) = 0$$

Degree is not defined.

2. Linear polynomial

$$p(x) = ax + b$$

Degree is 1.

3. Quadratic polynomial

$$p(x) = ax^2 + bx + c$$

Degree is 2.

4. Cubic polynomial

$$p(x) = ax^3 + bx^2 + cx + d$$

Degree is 3.

1.4 Monomials, Binomials and Trinomials

1. Monomial has one term, example

$$5x$$

2. Binomial has two terms, example

$$x + 3$$

3. Trinomial has three terms, example

$$x^2 + 2x + 1$$

1.5 Addition and Subtraction of Polynomials

1. Addition

$$(2x^2 + 3x + 1) + (x^2 - x + 4) = 3x^2 + 2x + 5$$

2. Subtraction

$$(2x^2 + 3x + 1) - (x^2 - x + 4) = x^2 + 4x - 3$$

Like terms are added or subtracted to simplify expressions.

1.6 Multiplication of Polynomials

- 1.

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

Each term of one polynomial is multiplied with every term of the other polynomial.

1.7 Division Algorithm for Polynomials

1. If $p(x)$ and $g(x)$ are polynomials with $g(x) \neq 0$, then

$$p(x) = g(x)q(x) + r(x)$$

where $q(x)$ is the quotient and $r(x)$ is the remainder.

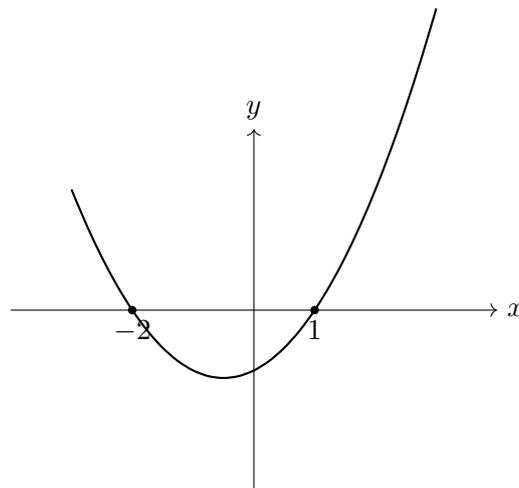
This is used to divide a polynomial by a linear divisor.

1.8 Zero of a Polynomial

1. A real number a is a zero of $p(x)$ if

$$p(a) = 0$$

Zeros represent the x-intercepts of the graph of the polynomial.



1.9 Remainder Theorem

1. If a polynomial $p(x)$ is divided by $(x - a)$, then the remainder is $p(a)$.

This theorem is used to quickly find remainders without actual division.

1.10 Factorization Using Identities

1.

$$(x + y)^2 = x^2 + 2xy + y^2$$

2.

$$(x - y)^2 = x^2 - 2xy + y^2$$

3.

$$x^2 - y^2 = (x + y)(x - y)$$

4.

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

These identities help in simplifying and factorizing polynomials.

1.11 Factorization of Trinomials

1.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

The trinomial is factorized by finding two numbers whose sum and product match the middle and constant terms.

1.12 Quick Revision Summary

1.

$$p(x) = a_n x^n + \dots + a_0$$

2.

$$p(x) = g(x)q(x) + r(x)$$

3.

$$p(a) = 0$$

4.

$$(x + y)^2 = x^2 + 2xy + y^2$$

5.

$$x^2 - y^2 = (x + y)(x - y)$$