SOLUTIONS

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Section A: Multiple Choice Questions (1 Mark Each)

1. Which of the following is an irrational number?

   (a) $\sqrt{225}$

   (b) 0.3796

   (c) 7.478478...

   (d) 1.1010010001...

   Answer: (d) 1.1010010001...

2. The value of $\left[(64)^{\frac{1}{2}}\right]^{\frac{1}{3}}$ is:

   (a) 2

   (b) 4

   (c) 8

   (d) 1

   Solution:

   $$\left[(64)^{\frac{1}{2}}\right]^{\frac{1}{3}} = [8]^{\frac{1}{3}} = 2$$

   Answer: (a) 2

3. Every rational number is:

   (a) A natural number

   (b) An integer

   (c) A real number

   (d) A whole number

   Answer: (c) A real number

4. Between two rational numbers, there are:

   (a) Exactly one rational number

   (b) Infinitely many rational numbers

   (c) Many irrational numbers only

   (d) No rational number

   Answer: (b) Infinitely many rational numbers

5. The product of any two irrational numbers is:

   (a) Always an irrational number

   (b) Always a rational number

   (c) Always an integer

   (d) Sometimes rational, sometimes irrational

   Answer: (d) Sometimes rational, sometimes irrational

6. The decimal representation of the rational number $\frac{33}{2^2 \times 5}$ is:

   (a) Terminating

(b) Non-terminating repeating

(c) Non-terminating non-repeating

(d) None of these

Solution:

$$\frac{33}{2^2 \times 5} = \frac{33}{20} = 1.65$$

The decimal representation is terminating. **Answer:** (a) Terminating

7. The value of $\sqrt[4]{\sqrt[3]{2^2}}$ is:

(a) $2^{-\frac{1}{6}}$

(b) 2^{-6}

(c) $2^{\frac{1}{6}}$

(d) 2^6

Solution:

$$\sqrt[4]{\sqrt[3]{2^2}} = \left(2^{\frac{2}{3}}\right)^{\frac{1}{4}} = 2^{\frac{2}{3} \times \frac{1}{4}} = 2^{\frac{1}{6}}$$

Answer: (c) $2^{\frac{1}{6}}$

8. If $x = 2 + \sqrt{3}$, then the value of $x + \frac{1}{x}$ is:

(a) 4

(b) $2\sqrt{3}$

(c) -4

(d) $2 - \sqrt{3}$

Solution:

$$x + \frac{1}{x} = (2 + \sqrt{3}) + \frac{1}{2 + \sqrt{3}} = (2 + \sqrt{3}) + \frac{2 - \sqrt{3}}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= (2 + \sqrt{3}) + \frac{2 - \sqrt{3}}{4 - 3} = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$$

Answer: (a) 4

Section B: Very Short Answer Questions (2 Marks Each)

1. Express $0.2\overline{35}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution: Let $x = 0.2\overline{35}$.

$$10x = 2.\overline{35}$$

$$1000x = 235.\overline{35}$$

Subtract the two equations:

$$1000x - 10x = 235.\overline{35} - 2.\overline{35}$$

$$990x = 233 \implies x = \frac{233}{990}$$

Answer: $\boxed{\dfrac{233}{990}}$

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2. Simplify: $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$.

Solution:

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} = \frac{2^3}{3^3} = \frac{8}{27}$$

$$\left(\frac{25}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{25}\right)^{\frac{3}{2}} = \left(\frac{3^2}{5^2}\right)^{\frac{3}{2}} = \frac{3^3}{5^3} = \frac{27}{125}$$

$$\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$$

$$\frac{27}{125} \div \frac{8}{125} = \frac{27}{8}$$

$$\frac{8}{27} \times \frac{27}{8} = 1$$

Answer: $\boxed{1}$

3. Rationalize the denominator of: $\frac{1}{\sqrt{7}-\sqrt{6}}$.

Solution:

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

Answer: $\boxed{\sqrt{7}+\sqrt{6}}$

4. Find two irrational numbers between 0.1 and 0.12.

Solution: Two irrational numbers between 0.1 and 0.12 are 0.1010010001... and 0.1101100110001....

Answer: $\boxed{0.1010010001..., 0.1101100110001...}$

Section C: Short Answer Questions (3 Marks Each)

1. If a and b are rational numbers and $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$, find the values of a and b.

Solution:

$$\frac{3+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{(3+\sqrt{7})^2}{9-7} = \frac{9+6\sqrt{7}+7}{2} = \frac{16+6\sqrt{7}}{2} = 8+3\sqrt{7}$$

Comparing with $a + b\sqrt{7}$, we get $a = 8$ and $b = 3$. **Answer:** $\boxed{a = 8, b = 3}$

2. Represent $\sqrt{3}$ on the number line using a geometric construction.

Solution:

(a) Draw a number line and mark the points $O(0)$ and $A(1)$ on it.

(b) At point A, draw a perpendicular AB of length $\sqrt{2}$.

(c) Join O to B. Then,

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}.$$

(d) With centre O and radius OB, draw an arc cutting the number line at point P.

(e) The point P represents $\sqrt{3}$ on the number line.
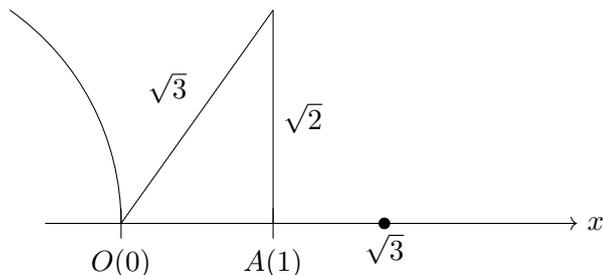
Answer: The point P on the number line represents $\sqrt{3}$.

3. Simplify the following expression: $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}}$.

   Solution: Rationalize each term:

   $$\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{1-2} = \sqrt{2}-1$$

   $$\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = \sqrt{3}-\sqrt{2}$$

   $$\frac{1}{\sqrt{3}+\sqrt{4}} \times \frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}} = \frac{\sqrt{3}-2}{3-4} = 2-\sqrt{3}$$

   Adding them together:

   $$(\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + (2-\sqrt{3}) = 1$$

   Answer: $\boxed{1}$

Section D: Long Answer / HOTS Questions (5 Marks Each)

1. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find the value of $x^2 + y^2 + xy$.

   Solution: Rationalize x and y:

   $$x = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} = 3+2\sqrt{6}+2 = 5+2\sqrt{6}$$

   $$y = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = 3-2\sqrt{6}+2 = 5-2\sqrt{6}$$

   $$x^2 = (5+2\sqrt{6})^2 = 25+20\sqrt{6}+24 = 49+20\sqrt{6}$$

   $$y^2 = (5-2\sqrt{6})^2 = 25-20\sqrt{6}+24 = 49-20\sqrt{6}$$

   $$xy = (5+2\sqrt{6})(5-2\sqrt{6}) = 25-24 = 1$$

   $$x^2 + y^2 + xy = (49+20\sqrt{6}) + (49-20\sqrt{6}) + 1 = 99$$

   Answer: $\boxed{99}$

2. Visualize the representation of 3.765 on the number line using successive magnification.

   Solution: The provided diagram is correct and accurately represents 3.765 on the number line using successive magnification. **Answer:** The point 3.765 is correctly marked on the number line as shown in the diagram.