Chapter 6: Lines and Angles

This chapter explores the fundamental relationships between lines and the angles they form. Understanding these relationships is crucial for all later work in geometry.

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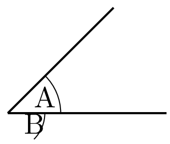
1. Basic Angle Pairs

When lines meet or cross, they create special pairs of angles with specific relationships.

1. **Complementary Angles**

$$\angle A + \angle B = 90^\circ$$

   - $\angle A, \angle B$: Two angles.
   - **Meaning**: Their measures add up to a right angle (90°).
   - **Usage**: If one angle is known, the other is $90^\circ -$ (known angle).

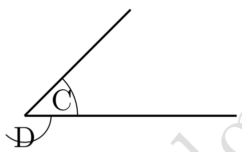


$\angle A + \angle B = 90^\circ$

2. **Supplementary Angles**

$$\angle C + \angle D = 180^\circ$$

   - $\angle C, \angle D$: Two angles.
   - **Meaning**: Their measures add up to a straight angle (180°).
   - **Usage**: If one angle is known, the other is $180^\circ -$ (known angle).

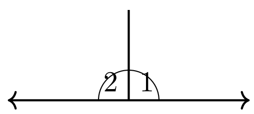


$\angle C + \angle D = 180^\circ$

3. **Linear Pair of Angles**

$$\angle 1 + \angle 2 = 180^\circ$$

   - $\angle 1, \angle 2$: Two adjacent angles formed when a ray stands on a line.
   - **Meaning**: They are adjacent and supplementary.
   - **Usage**: A very common result used in proofs. If angles form a linear pair, they are supplementary.



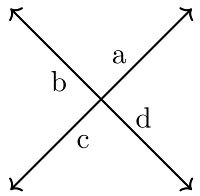
Adjacent + Supplementary

4. **Vertically Opposite Angles**

$$\angle a = \angle c \quad \text{and} \quad \angle b = \angle d$$

   - $\angle a, \angle b, \angle c, \angle d$: Angles formed by two intersecting lines.

- **Meaning**: Angles opposite each other at the intersection are equal.
- **Usage**: A key property for solving problems involving intersecting lines.



$\angle a = \angle c$ and $\angle b = \angle d$

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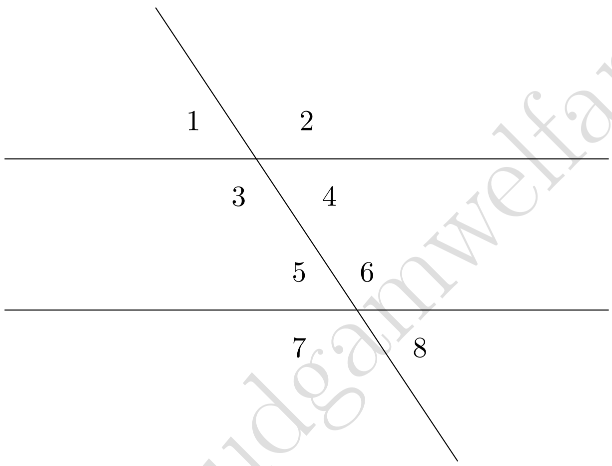
2. Angles Made by a Transversal

When a line (transversal) crosses two other lines, it creates eight angles with special relationships, especially when the lines are parallel.

Angle Properties When a Transversal Intersects Two Parallel Lines



1. **Corresponding Angles** If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

$$\angle 1 = \angle 5, \quad \angle 2 = \angle 6, \quad \angle 3 = \angle 7, \quad \angle 4 = \angle 8$$

2. **Alternate Interior Angles** If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$$\angle 3 = \angle 6, \quad \angle 4 = \angle 5$$

3. **Alternate Exterior Angles** If a transversal intersects two parallel lines, then each pair of alternate exterior angles is equal.

$$\angle 1 = \angle 8, \quad \angle 2 = \angle 7$$

4. **Interior Angles on the Same Side of the Transversal (Co-interior Angles)** The pair of interior angles on the same side of the transversal are supplementary.

$$\angle 3 + \angle 5 = 180^\circ, \quad \angle 4 + \angle 6 = 180^\circ$$

5. **Vertically Opposite Angles** When two lines intersect, vertically opposite angles are equal.

$$\angle 1 = \angle 4, \quad \angle 2 = \angle 3, \quad \angle 5 = \angle 8, \quad \angle 6 = \angle 7$$

6. **Linear Pair of Angles** A pair of adjacent angles forming a straight line is supplementary.

$$\angle 1 + \angle 2 = 180^\circ, \quad \angle 3 + \angle 4 = 180^\circ$$

7. **Converse of Corresponding Angles Axiom** If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel.

8. **Converse of Alternate Interior Angles Theorem** If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

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3. Parallel Line Properties

These theorems help determine when lines are parallel and establish relationships between parallel lines.

8. **Condition for Parallel Lines (Converse of Corresponding Angles)**

$$\text{If } \angle 1 = \angle 5, \text{ then } l \parallel m$$

   - **Usage**: If corresponding angles are equal, then the lines are parallel.

9. **Condition for Parallel Lines (Converse of Alternate Angles)**

$$\text{If } \angle 3 = \angle 5, \text{ then } l \parallel m$$

   - **Usage**: If alternate interior angles are equal, then the lines are parallel.

10. **Condition for Parallel Lines (Converse of Co-interior Angles)**

$$\text{If } \angle 3 + \angle 6 = 180^\circ, \text{ then } l \parallel m$$

   - **Usage**: If interior angles on the same side are supplementary, then the lines are parallel.

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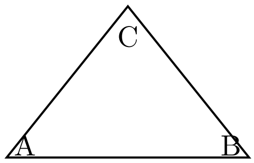
4. Triangle Angle Properties

These fundamental properties relate to the angles of a triangle.

11. **Angle Sum Property of a Triangle**

$$\angle A + \angle B + \angle C = 180°$$

- $\angle A, \angle B, \angle C$: The three interior angles of any triangle.
- **Meaning**: The sum of all interior angles of a triangle is always 180°.
- **Usage**: To find the third angle when two angles are known, or to check if three angles can form a triangle.

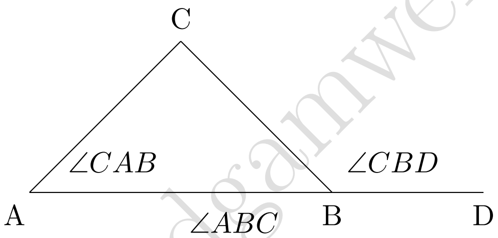


$\angle A + \angle B + \angle C = 180°$

12. **Exterior Angle Property of a Triangle**

$$\angle CBD = \angle CAB + \angle ABC$$

- $\angle CBD$ is the exterior angle formed by extending side BC to point D.
- $\angle CAB$ and $\angle ABC$ are the two interior opposite angles.
- An exterior angle of a triangle is equal to the sum of its two interior opposite angles.



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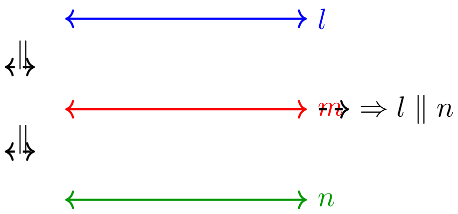
5. Lines Parallel to the Same Line

This important theorem establishes the transitivity of parallelism.

13. **Lines Parallel to the Same Line**

$$\text{If } l \parallel m \text{ and } m \parallel n, \text{ then } l \parallel n$$

- **Usage**: If two lines are each parallel to a third line, then they are parallel to each other.



Quick Revision Summary

Here are all the essential relationships and formulas from this chapter.

1. **Complementary Angles**: $\angle A + \angle B = 90^\circ$
2. **Supplementary Angles**: $\angle C + \angle D = 180^\circ$
3. **Linear Pair**: $\angle 1 + \angle 2 = 180^\circ$ (adjacent angles on straight line)
4. **Vertically Opposite Angles**: $\angle a = \angle c$, $\angle b = \angle d$
5. **Corresponding Angles (parallel lines)**: $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$
6. **Alternate Interior Angles (parallel lines)**: $\angle 3 = \angle 5$, $\angle 4 = \angle 6$
7. **Interior Angles on Same Side (parallel lines)**: $\angle 3 + \angle 6 = 180^\circ$, $\angle 4 + \angle 5 = 180^\circ$
8. **Angle Sum Property of Triangle**: $\angle A + \angle B + \angle C = 180^\circ$
9. **Exterior Angle Property**: $\angle ACD = \angle A + \angle B$
10. **Lines Parallel to Same Line**: If $l \parallel m$ and $m \parallel n$, then $l \parallel n$