Chapter 12: Heron's Formula

This chapter introduces a powerful formula to calculate the area of a triangle when all three side lengths are known. Unlike the standard formula ($\frac{1}{2} \times$ base $\times$ height), Heron's formula doesn't require you to know the height of the triangle.

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1. Heron's Formula for Area of Triangle

This is the main formula of the chapter, named after the Greek mathematician Heron of Alexandria.

1. **Heron's Formula**

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

- a, b, c: Lengths of the three sides of the triangle.
- s: Semi-perimeter of the triangle.
- $s = \frac{a+b+c}{2}$: Half of the triangle's perimeter.
- **Usage**: Use when you know all three sides but not the height. Follow these steps:
  - (a) Calculate semi-perimeter $s = \frac{a+b+c}{2}$
  - (b) Substitute s, a, b, c into the formula
  - (c) Compute the product under the square root
  - (d) Take the square root for the area
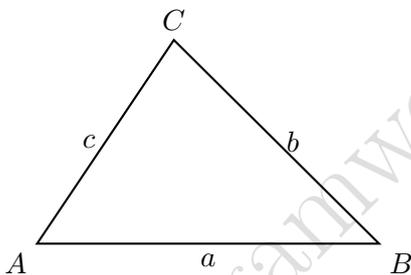
$$s = \frac{a+b+c}{2}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

2. **Semi-perimeter Formula**

$$s = \frac{a+b+c}{2}$$

- **Usage**: Always calculate this first before applying Heron's formula.

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2. Alternative Form of Heron's Formula

Sometimes it's useful to see the expanded form.

3. **Expanded Form**

$$\text{Area} = \frac{1}{4}\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}$$

- **Note**: This is algebraically equivalent to the main formula. The expression under the square root is the same as $16s(s-a)(s-b)(s-c)$.
- **Usage**: Some problems may present the formula in this form.

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3. Special Case: Equilateral Triangle

For an equilateral triangle (all sides equal), Heron's formula simplifies beautifully.

4. **Area of Equilateral Triangle**

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

- a: Length of each side of the equilateral triangle.
- **Derivation**: When $a = b = c$, then $s = \frac{3a}{2}$, and Heron's formula simplifies to this result.
- **Usage**: Direct formula for equilateral triangles, much faster than using general Heron's formula.

a a

a

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

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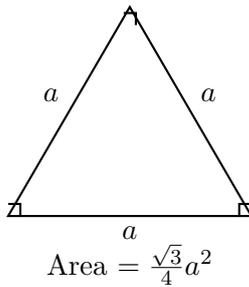
4. Application to Quadrilaterals

Heron's formula can be extended to find areas of certain quadrilaterals by dividing them into triangles.

5. **Area of Quadrilateral (when diagonal and sides are known)**

$$\text{Area} = \text{Area}(\triangle ABC) + \text{Area}(\triangle ADC)$$

- **Method**: Divide quadrilateral into two triangles using a diagonal.
- **Usage**: For quadrilaterals where you can measure all sides and one diagonal.

D

C

$\triangle ADC$

$\triangle ABC$

A B

$$\text{Area} = \text{Area}(\triangle ABC) + \text{Area}(\triangle ADC)$$
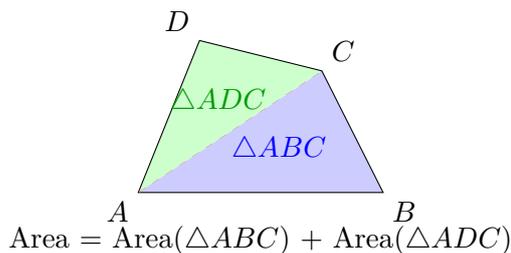
6. **For Cyclic Quadrilateral (Brahmagupta's Formula - Extension)**

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- a, b, c, d: Sides of cyclic quadrilateral (all vertices lie on a circle).
- $s = \frac{a+b+c+d}{2}$: Semi-perimeter.
- **Note**: This is mentioned for advanced students. It's Heron's formula extended to cyclic quadrilaterals.

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5. Step-by-Step Application Guide

Here's how to systematically apply Heron's formula to solve problems.

7. **Problem-Solving Steps**

(a) Identify the three sides a, b, c.

(b) Calculate semi-perimeter: $s = \frac{a+b+c}{2}$.

(c) Compute differences: $(s-a)$, $(s-b)$, $(s-c)$.

(d) Multiply: $s \times (s-a) \times (s-b) \times (s-c)$.

(e) Take square root of the product.

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6. Real-Life Applications

Heron's formula is useful in practical situations where direct height measurement is difficult.

8. **Common Applications**

- **Land measurement**: Finding area of triangular plots.
- **Construction**: Calculating material needed for triangular structures.
- **Navigation**: Determining areas from distance measurements.
- **Surveying**: When heights are inaccessible but sides can be measured.
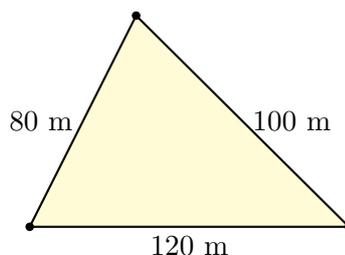
80 m   100 m

120 m

Triangular field with sides: 80m, 100m, 120m
Find area using Heron's Formula

9. **Example: Triangular Field**

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7. Important Notes and Tips

10. **Triangle Inequality Check**

    - Before using Heron's formula, verify that the given sides can form a triangle:

    $$a + b > c, \quad b + c > a, \quad c + a > b$$

    - If this fails, the sides don't form a valid triangle.

11. **Units**

    - Ensure all sides are in the same units before calculation.
    - Area will be in square units of the side measurement.

12. **Decimal vs Exact Form**

    - Keep calculations exact as long as possible.
    - Round only at the final answer if required.

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Quick Revision Summary

Here are all the essential formulas from this chapter.

1. **Semi-perimeter**: $s = \frac{a+b+c}{2}$

2. **Heron's Formula**: Area $= \sqrt{s(s-a)(s-b)(s-c)}$

3. **Equilateral Triangle**: Area $= \frac{\sqrt{3}}{4}a^2$

4. **Quadrilateral Area**: Area $= \text{Area}(\triangle ABC) + \text{Area}(\triangle ADC)$

5. **Triangle Inequality**: $a + b > c$, $b + c > a$, $c + a > b$