

Chapter 5: Introduction to Euclid's Geometry

This chapter introduces you to the foundational thinking of geometry as developed by the ancient Greek mathematician Euclid. You will learn about the building blocks of logical reasoning in mathematics: definitions, axioms, and postulates.

1. Fundamental Building Blocks

Geometry starts with simple, accepted truths from which complex results (theorems) are logically derived.

1. A Definition

- A **definition** is a precise statement that explains the meaning of a new term using already known terms.
- **Example:** "A point is that which has no part." or "A line is breadthless length."
Usage: Definitions create a common language for geometry.

2. An Axiom or a Postulate

Axiom/Postulate = A statement accepted as true without proof

- **Axioms** are common notions accepted in all sciences.
- **Postulates** are specific assumptions for geometry.
- **Usage:** These are the starting points for all proofs. You must remember and apply them.

3. A Theorem

Theorem = A statement proved using definitions, axioms, postulates, and previously proved theorems

- **Usage:** The goal is to logically deduce theorems from the basic assumptions.

2. Euclid's Five Postulates

These are the five foundational assumptions on which Euclid built his entire geometry.

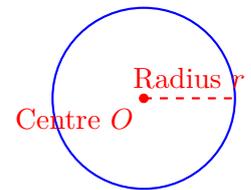
4. Postulate 1: A straight line can be drawn from any point to any other point.

- Given any two distinct points, there is a unique line that passes through them.



5. Postulate 2: A terminated line (line segment) can be produced indefinitely in both directions.

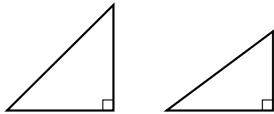
- A line segment can be extended to form a full line.



6. **Postulate 3:** A circle can be drawn with any centre and any radius.

7. **Postulate 4:** All right angles are equal to one another.

- A right angle is a standard measure. Every right angle measures 90° .



Right Angle Right Angle

8. **Postulate 5 (The Famous Parallel Postulate):** *If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.*

3. Key Axioms (Common Notions)

These are general logical principles used throughout mathematics.

9. Axioms of Equality and Magnitude

- **Things which are equal to the same thing are equal to one another.** (If $a = c$ and $b = c$, then $a = b$).
- **If equals are added to equals, the wholes are equal.**
- **If equals are subtracted from equals, the remainders are equal.**
- **Things which coincide with one another are equal to one another.**
- **The whole is greater than the part.**

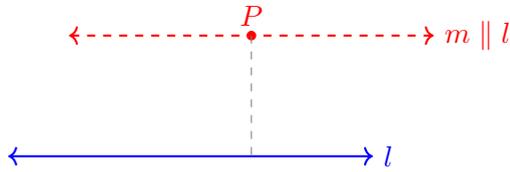
4. Equivalent Versions of Euclid's Fifth Postulate

Because the original 5th postulate was complex, mathematicians searched for simpler, logically equivalent statements. The most famous is ****Playfair's Axiom****.

10. Playfair's Axiom

For every line l and for every point P not lying on l , there exists a unique line m passing through P and

- l : A given straight line.
- P : A point not on l .
- m : The unique line through P parallel to l .
- **Usage:** This is the modern form of the parallel postulate used in most school geometries. It's much easier to understand and apply.



5. Some Important Theorems/Results

These are simple yet powerful results deduced from the axioms and postulates.

11. Two distinct lines cannot have more than one point in common.

- If two lines share two points, they are the same line (from Postulate 1).



12. Two distinct intersecting lines cannot be parallel to the same line.

Impossible if l_1 and l_3 intersect

Quick Revision Summary

Here are the core ideas and statements from this chapter for a quick recap.

1. **Definitions** explain the meaning of new terms.
2. **Axioms/Postulates** are assumptions accepted without proof.
3. **Theorems** are statements that are proved.
4. **Postulate 1:** A unique line through any two points.
5. **Postulate 2:** A line segment can be extended indefinitely.
6. **Postulate 3:** A unique circle for any centre and radius.
7. **Postulate 4:** All right angles are equal.
8. **Postulate 5 (Parallel Postulate):** If interior angles sum to $< 180^\circ$, lines meet on that side.

9. **Playfair's Axiom (Equivalent to Postulate 5):**

Through a point not on a line, there is exactly one line parallel to the given line.

10. **Common Axioms:** Things equal to the same thing are equal; the whole is greater than the part.

www.udgamwelfarefoundation.com