CHAPTER TEST: INTRODUCTION TO EUCLID'S GEOMETRY

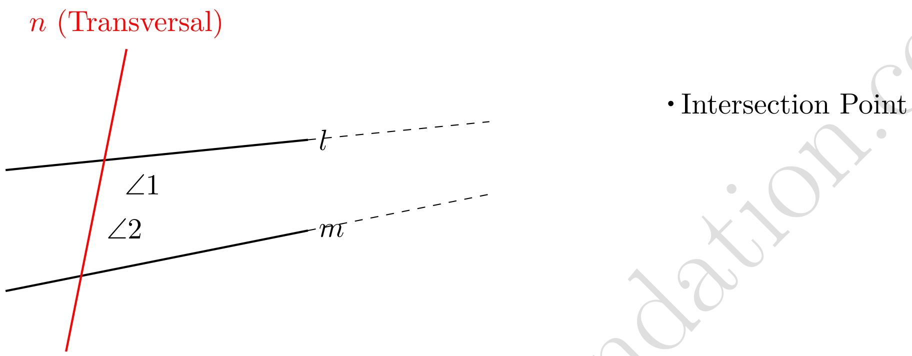
Mathematics | Class IX (2026/EUCLID/09/LongAns/002)

Time: 1 Hour **Max. Marks: 25**

LONG ANSWER QUESTIONS: EUCLID'S GEOMETRY
SET – 2 (The Fifth Postulate and its Equivalents)

Q1. Euclid's Fifth Postulate: If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles (180°), then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Illustration:



In the diagram, if $\angle 1 + \angle 2 < 180°$, lines l and m will eventually intersect on the right side.

Q2. Playfair's Axiom: For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.

Equivalence: Euclid's 5th postulate implies that if the sum of angles is *exactly* 180°, the lines will never meet (parallel). Playfair's Axiom simplifies this by stating only *one* such parallel line can exist through a specific point. Both statements lead to the same logical conclusion regarding the nature of parallel lines in a flat plane.

Mathematical Quest: Mathematicians tried to prove the 5th postulate because it was much more complex than the other four. They believed it was a theorem that could be derived from the simpler postulates 1–4. This quest eventually led to the discovery of Non-Euclidean geometries.

Q3. Proof:

- Let l, m, and n be three lines. Let l and m be two distinct intersecting lines.
- Suppose both l and m are parallel to line n.
- Let the point of intersection of l and m be P.
- This means that through a point P (not on line n), there are **two** distinct lines (l and m) parallel to line n.
- However, this directly contradicts **Playfair's Axiom**, which states that through a point not on a line, there is only **one unique** line parallel to the given line.
- Therefore, our assumption is wrong. **Conclusion:** Two distinct intersecting lines cannot be parallel to the same line.

Q4. Analysis of Postulates:

(i) **Undefined Terms:** Both postulates contain undefined terms such as **'point'**, **'line'**, and **'in between'**.

(ii) **Consistency:** Yes, they are consistent. They do not contradict each other; it is possible for a point to exist between two others while also having a third point not on that same line.

(iii) **Derivation from Euclid:**

- Postulate (i) is a consequence of the fact that lines are continuous and infinitely divisible (related to Postulate 2).
- Postulate (ii) is an independent assumption that defines the existence of a 2D plane. Euclid assumes this implicitly in his constructions (like the equilateral triangle), but it is not explicitly stated in his original five postulates.

Q5. Consistency in Axioms: A system of axioms is said to be **consistent** if it is impossible to derive a contradiction from them. If a system is inconsistent, any statement (both true and false) could be proven, making the system mathematically useless.

Contradiction Example: If we added a postulate saying "Two parallel lines eventually meet," it would contradict the definition of parallel lines in Euclidean geometry.

Historical Context: For centuries, mathematicians tried to replace Euclid's 5th Postulate. By assuming the 5th Postulate was *false* (e.g., assuming multiple parallel lines or no parallel lines exist), they didn't find a contradiction. Instead, they discovered **Non-Euclidean Geometries** (like Hyperbolic and Elliptic geometry). This proved that Euclid's system was consistent but not the *only* possible geometric system.