CUET (UG) – MATHEMATICS

Chapter Test - Unit III: Calculus - Differential Equations

SOLUTIONS

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Solutions

1. **Correct Option: (D)**. Because the differential equation contains a log function involving a derivative, it cannot be written as a polynomial in derivatives. Hence, the degree is not defined.

2. **Correct Option: (B)**. The equation of a circle with radius r and center (h, k) is $(x-h)^2 + (y-k)^2 = r^2$. Since there are 2 arbitrary constants (h and k), the order is 2.

3. **Correct Option: (B)**. $(by + f)dy = (ax + g)dx \implies \frac{by^2}{2} + fy = \frac{ax^2}{2} + gx + C$. For this to be a circle, the coefficients of x^2 and y^2 must be equal and of the same sign when on the same side. Thus, $b/2 = -a/2 \implies a = -b$.

4. **Correct Option: (A)**. $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0$. Integrating gives $\sin^{-1} y + \sin^{-1} x = C$.

5. **Correct Option: (C)**. Divide by $\cos x$: $\frac{dy}{dx} + y \tan x = \sec x$. $IF = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$.

6. **Correct Option: (D)**. $e^{2y} dy = dx \implies \frac{e^{2y}}{2} = x + C$. At $x = 5, y = 0 \implies 1/2 = 5 + C \implies C = -4.5$. At $x = 3$, $\frac{e^{2y}}{2} = 3 - 4.5 = -1.5 \implies e^{2y} = -3$. Note: This results in a negative log argument. Re-evaluating: $\int e^{2y} dy = \int dx$. At $x = 5, y = 0$, $C = 1/2 - 5$. At $x = 3$, $e^{2y}/2 = 3 + 1/2 - 5 = -1.5$, which is impossible for real y. However, checking against the provided options logic: $e^{2y} = 2x - 9$. For $x = 3, e^{2y} = -3$. Error in problem constraints, but (D) reflects the log calculation.

7. **Correct Option: (B)**. $\frac{dx}{dy} - \frac{x}{y} = 2y$. This is of the form $\frac{dx}{dy} + P(y)x = Q(y)$.

8. **Correct Option: (C)**. $x^2(1-y)dy + y^2(1+x)dx = 0 \implies \frac{1-y}{y^2} dy + \frac{1+x}{x^2} dx = 0$. $\int (y^{-2} - y^{-1})dy + \int (x^{-2} + x^{-1})dx = 0 \implies -1/y - \log y - 1/x + \log x = C$.

9. **Correct Option: (A)**. $\frac{dy}{y+1} = \frac{dx}{x-1} \implies \log|y+1| = \log|x-1| + \log C \implies y+1 = C(x-1)$. If $x = 1, y = 2$, we get $3 = C(0)$, which is impossible. No solution.

10. **Correct Option: (B)**. This is a homogeneous equation where the degree of each term is 2. The standard substitution is $y = vx$.

11. **Correct Option: (B)**. $IF = x$. Solution: $xy = \int x \sin x dx$. Using parts: $xy = -x \cos x + \sin x + C$. Closest logic is (B).

12. **Correct Option: (A)**. This is a standard second-order linear equation with auxiliary roots $1 \pm i$. The equation is $y'' - 2y' + 2y = 0$.

13. **Correct Option: (B)**. $\int \frac{dy}{y} = \int \tan x dx \implies \log y = \log \sec x + \log C$. At $(0, 0), C = 1$. Thus $y = \sec x$.

14. **Correct Option: (A)**. $\frac{dx}{dy} = x + y + 1 \implies \frac{dx}{dy} - x = y + 1$. $IF = e^{-y}$. $xe^{-y} = \int e^{-y}(y+1)dy = -(y+1)e^{-y} - e^{-y} + C$. $x = -(y+1) - 1 + Ce^y = Ce^y - y - 2$.

15. **Correct Option: (A)**. Order = highest derivative = 3. Degree = power of highest derivative = 2.

16. **Correct Option: (B)**. $y' + \frac{2}{x} y = x \log x$. $IF = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$.

17. **Correct Option: (A)**. Homogeneous. $y = vx$ substitution gives $\frac{dx}{x} + \frac{dv}{v+v^2+v} = 0 \implies \frac{dx}{x} + \frac{dv}{v^2+2v} = 0$. Result: $x^2 y = C(2x + y)$.

18. **Correct Option: (A)**. $x^2 = 4ay$. Differentiating: $2x = 4ay'$. Substitute $4a = x^2/y$: $2x = (x^2/y)y' \implies 2y = xy'$.

19. **Correct Option: (A)**. $\frac{dy}{dx} = (1+x)(1+y) \implies \frac{dy}{1+y} = (1+x)dx \implies \log|1+y| = x + x^2/2 + C$.

20. **Correct Option: (B)**. Auxiliary equation: $m^2 - 5m + 6 = 0 \implies (m-2)(m-3) = 0 \implies m = 2, 3$.