CUET (UG) – MATHEMATICS

Chapter Test - Integration and its Applications

SOLUTIONS

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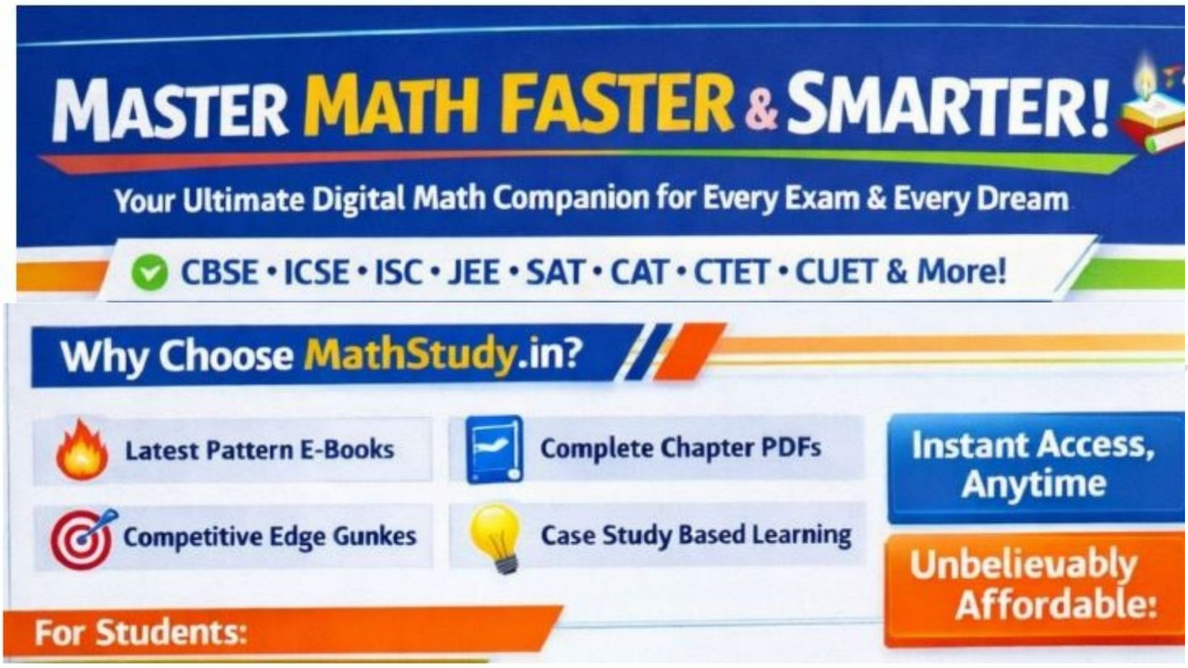

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Solutions

1. $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$. Using the formula $\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a})$, we get $\tan^{-1}(x+1)+C$. Correct Option: **(A)**

2. Using the property $\int e^x[f(x)+f'(x)]dx = e^x f(x)+C$. Here $f(x)=\sin x$ and $f'(x)=\cos x$. Result: $e^x \sin x + C$. Correct Option: **(B)**

3. Let $x^3 = t \implies 3x^2dx = dt$. The integral becomes $\int \frac{dt}{t^2+1} = \tan^{-1}(t) = \tan^{-1}(x^3)+C$. Thus $f(x)=x^3$. Correct Option: **(C)**

4. Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$. Using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$. Adding both gives $2I = \int_0^{\pi/2} 1dx = \pi/2 \implies I = \pi/4$. Correct Option: **(C)**

5. Area $= \int_1^2 x^2dx = [\frac{x^3}{3}]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ sq. units. Correct Option: **(A)**

6. Let $1+\log x = t \implies \frac{1}{x}dx = dt$. Integral becomes $\int \frac{1}{t^2}dt = -t^{-1} = \frac{-1}{1+\log x}+C$. Correct Option: **(A)**

7. Area $= \int_0^{\pi} |\cos x|dx = \int_0^{\pi/2} \cos x dx + \int_{\pi/2}^{\pi}(-\cos x)dx = [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} = (1-0)-(0-1) = 2$ sq. units. Correct Option: **(C)**

8. $\int_{-1}^1 |x|dx = 2\int_0^1 xdx$ (since $|x|$ is even) $= 2[\frac{x^2}{2}]_0^1 = 1$. Correct Option: **(B)**

9. $\int \frac{\cos^2 x - \sin^2 x}{(\sin x+\cos x)^2}dx = \int \frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}dx = \int \frac{\cos x-\sin x}{\cos x+\sin x}dx$. Let $\sin x+\cos x = t \implies (\cos x - \sin x)dx = dt$. Integral is $\log|t| = \log|\sin x+\cos x|+C$. Correct Option: **(A)**

10. By Leibniz Rule (Fundamental Theorem of Calculus), $\frac{d}{dx}\int_0^x f(t)dt = f(x)$. So $f'(x) = x\sin x$. Correct Option: **(C)**

11. $y = \sqrt{a^2-x^2}$. Area $= 4\int_0^a \sqrt{a^2-x^2}dx = 4[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}]_0^a = 4[0+\frac{a^2}{2}\cdot\frac{\pi}{2}] = \pi a^2$. Correct Option: **(C)**

12. $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{4((\frac{3}{2})^2-x^2)}} = \frac{1}{2}\int \frac{dx}{\sqrt{(\frac{3}{2})^2-x^2}} = \frac{1}{2}\sin^{-1}(\frac{x}{3/2}) = \frac{1}{2}\sin^{-1}(\frac{2x}{3})+C$. Correct Option: **(B)**

13. $\int_0^{\pi} \frac{1-\cos 2x}{2}dx = [\frac{1}{2}x - \frac{\sin 2x}{4}]_0^{\pi} = \frac{\pi}{2}$. Correct Option: **(A)**

14. Area $= \int_1^e \log x dx = [x\log x - x]_1^e = (e\log e - e) - (1\log 1 - 1) = (e-e)-(0-1) = 1$ sq. unit. Correct Option: **(B)**

15. Divide numerator and denominator by e^x: $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$. Let $e^x+e^{-x} = t \implies (e^x-e^{-x})dx = dt$. Result: $\log|e^x+e^{-x}|+C$. Correct Option: **(A)**

16. Let $x^2+1 = t \implies 2xdx = dt \implies xdx = dt/2$. $\frac{1}{2}\int_5^{10}\frac{dt}{t} = \frac{1}{2}[\log t]_5^{10} = \frac{1}{2}(\log 10 - \log 5) = \frac{1}{2}\log(2)$. Correct Option: **(C)**

17. Using Integration by parts, $\int \sec^3 xdx = \frac{1}{2}(\sec x\tan x + \log|\sec x+\tan x|)+C$. It involves both. Correct Option: **(C)**

18. $y = 2\sqrt{x}$. Area $= 2\int_0^3 2\sqrt{x}dx = 4[\frac{x^{3/2}}{3/2}]_0^3 = \frac{8}{3}[3\sqrt{3}] = 8\sqrt{3}$ sq. units. Correct Option: **(B)**

19. $[\tan^{-1} x]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \pi/4$. Correct Option: **(A)**

20. Let $\cos x = t \implies -\sin xdx = dt$. Integral is $\int \frac{-dt}{1+t^2} = -\tan^{-1}(t) = -\tan^{-1}(\cos x)+C$. Correct Option: **(B)**

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