CUET (UG) – MATHEMATICS

Chapter Test -Calculus (Higher Order Derivatives, Increasing/Decreasing, Maxima/Minima)

SOLUTIONS

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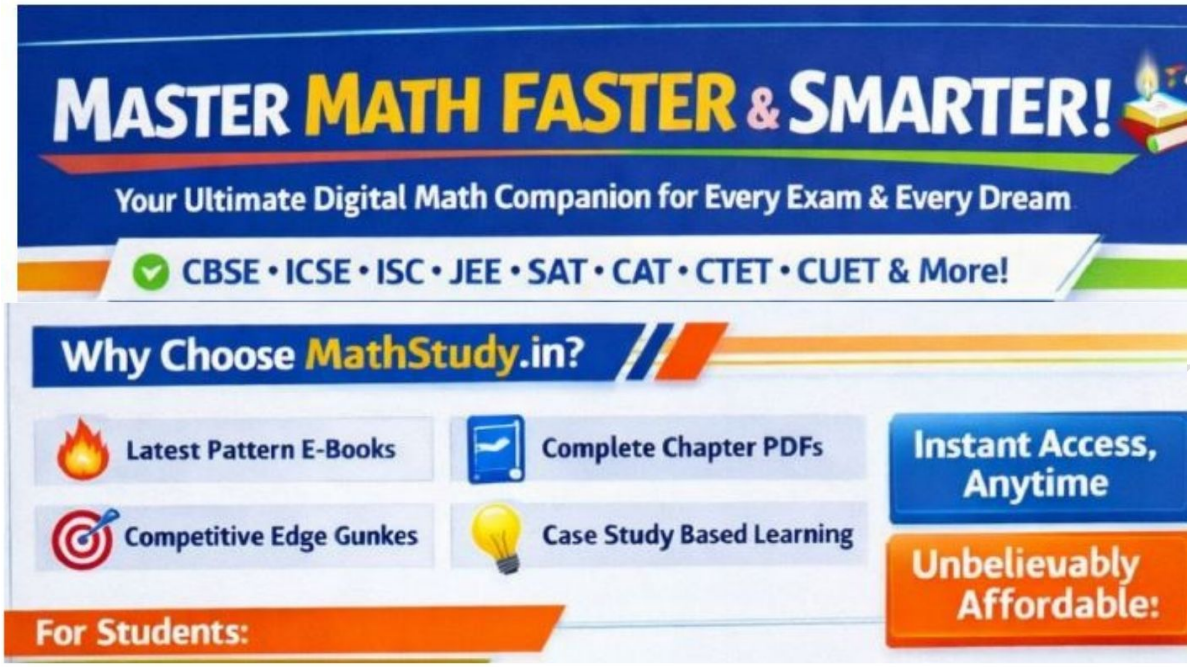

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Solutions

1. $y = \log(\sin x) \implies y' = \frac{1}{\sin x} \cdot \cos x = \cot x$. Differentiating again, $y'' = -\text{cosec}^2 x$. Correct Option: **(A)**

2. $f'(x) = 2x - 4$. For strictly increasing, $f'(x) > 0 \implies 2x > 4 \implies x > 2$. Interval: $(2, \infty)$. Correct Option: **(B)**

3. $f(x) = \sin x + \cos x$. Maximum value occurs when $f'(x) = \cos x - \sin x = 0 \implies x = \pi/4$. $f(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$. Correct Option: **(C)**

4. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$. $\frac{d^2y}{dx^2} = \frac{d}{d\theta}(-\tan\theta) \cdot \frac{d\theta}{dx} = -\sec^2\theta \cdot \frac{1}{-3a\cos^2\theta\sin\theta}$. At $\theta = \pi/4$, $\cos\theta = \sin\theta = 1/\sqrt{2}$. $y'' = (2) \cdot \frac{1}{3a(1/2)(1/\sqrt{2})} = \frac{4\sqrt{2}}{3a}$. Correct Option: **(A)**

5. $f'(x) = \frac{\log x(1) - x(1/x)}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$. For $f'(x) > 0$, $\log x > 1 \implies x > e$. Correct Option: **(B)**

6. Using AM $\geq$ GM: $\frac{ax + b/x}{2} \geq \sqrt{ax \cdot \frac{b}{x}} = \sqrt{ab}$. So, $ax + b/x \geq 2\sqrt{ab}$. Correct Option: **(B)**

7. $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$. Critical points are $x = 1, 3$. $f''(x) = 6x - 12$. $f''(1) = -6 < 0$ (Maxima). $f(1) = 1 - 6 + 9 + 15 = 19$. Correct Option: **(A)**

8. Let $y = x^x \implies \ln y = x \ln x$. $\frac{1}{y}y' = 1 + \ln x \implies y' = x^x(1 + \ln x)$. $y' = 0 \implies \ln x = -1 \implies x = 1/e$. Correct Option: **(B)**

9. $y' = 5Ae^{5x} - 5Be^{-5x} \implies y'' = 25Ae^{5x} + 25Be^{-5x} = 25y$. Correct Option: **(C)**

10. $f'(x) = -\sin x$. Strictly decreasing $\implies f'(x) < 0 \implies \sin x > 0$. In $(0, \pi)$, $\sin x$ is positive. Correct Option: **(A)**

11. Let point be (x, x^2). Square of distance $D = x^2 + (x^2 - 5)^2 = x^2 + x^4 - 10x^2 + 25 = x^4 - 9x^2 + 25$. $D' = 4x^3 - 18x = 0 \implies x^2 = 4.5 \implies y = 4.5$. Point is $(\sqrt{4.5}, 4.5)$. Correct Option: **(C)**

12. $f'(x) = k - \cos x$. For $f'(x) \geq 0$, $k \geq \cos x$. Max value of $\cos x$ is 1, so $k \geq 1$. Correct Option: **(C)**

13. $y = e^x \sin x \implies y' = e^x(\sin x + \cos x)$. $y'' = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = 2e^x \cos x$. Correct Option: **(A)**

14. $f'(x) = 3x^2 - 6x + 3 = 3(x-1)^2$. Since $f'(x) \geq 0$ for all x, $f(x)$ is increasing everywhere. Correct Option: **(A)**

15. A rectangle inscribed in a circle has max area when it is a square. Side $s = \sqrt{2}r$. Area $s^2 = 2r^2$. Correct Option: **(B)**

16. $y = \tan^{-1} x \implies y' = \frac{1}{1+x^2} \implies y'(1+x^2) = 1$. Differentiating again: $y''(1+x^2) + y'(2x) = 0$. Correct Option: **(B)**

17. $f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$. $f'(x) < 0$ for $x \in (2, 3)$. Correct Option: **(A)**

18. $x + y = 10 \implies y = 10 - x$. $S = x^2 + (10-x)^2$. $S' = 2x - 2(10-x) = 4x - 20 = 0 \implies x = 5$. $S = 25 + 25 = 50$. Correct Option: **(B)**

19. By definition, if the second derivative is negative, the function is concave downwards (or "concave"). Correct Option: **(B)**

20. $y' = 3x^2 - 3 = 0 \implies x = \pm 1$. In $[0, 2]$, check $x = 0, 1, 2$. $f(0) = 2$, $f(1) = 1 - 3 + 2 = 0$, $f(2) = 8 - 6 + 2 = 4$. Absolute max is 4. Correct Option: **(A)**