SOLUTIONS

CUET Mathematics Test

Chapter: Algebra (Matrices and Determinants)

Solutions

1. A square matrix where $a_{ij} = 0$ for $i \neq j$ is a diagonal matrix. If all diagonal elements are equal to a constant k, it is a Scalar matrix. Correct Option: **(B)**
2. $(I + A)^3 - 7A = I^3 + 3I^2A + 3IA^2 + A^3 - 7A$. Since $A^2 = A$, then $A^3 = A \cdot A^2 = A$. $I + 3A + 3A + A - 7A = I + 7A - 7A = I$. Correct Option: **(C)**
3. For $(A^{23} + B^{23})^T = (A^T)^{23} + (B^T)^{23}$. Since A, B are skew-symmetric, $A^T = -A, B^T = -B$. $(-A)^{23} + (-B)^{23} = -A^{23} - B^{23} = -(A^{23} + B^{23})$. Thus, it is skew-symmetric. Correct Option: **(D)**
4. $|A^3| = |A|^3 = 125 \implies |A| = 5$. $|A| = \alpha^2 - 4 = 5 \implies \alpha^2 = 9 \implies \alpha = \pm 3$. Correct Option: **(A)**
5. $|adj(A)| = |A|^{n-1} = 5^{3-1} = 5^2 = 25$. Correct Option: **(C)**
6. Matrix has 9 elements. Each has 2 choices. $2^9 = 512$. Correct Option: **(D)**
7. Using Cayley-Hamilton theorem, characteristic equation is $\lambda^2 - Tr(A)\lambda + |A| = 0$. $\lambda^2 - 5\lambda + (4 - 6) = 0 \implies A^2 - 5A - 2I = 0 \implies A^2 - 5A = 2I$. Correct Option: **(A)**
8. $(AB - BA)^T = (AB)^T - (BA)^T = B^TA^T - A^TB^T = BA - AB = -(AB - BA)$. Correct Option: **(B)**
9. $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$. So $A^4 = I^2 = I$. Correct Option: **(B)**
10. For no solution, $\Delta = 0$. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & k \end{vmatrix} = 0 \implies k = 5$. Correct Option: **(B)**
11. Property: $|adj(adj(A))| = |A|^{(n-1)^2}$. For $n = 3$, $(3 - 1)^2 = 4$. Correct Option: **(B)**
12. $A^{-1} = \frac{1}{|A|} adj(A)$. $|A| = 1$. $adj(A) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. Correct Option: **(A)**
13. Possible orders for 12 elements: $1 \times 12, 12 \times 1, 2 \times 6, 6 \times 2, 3 \times 4, 4 \times 3$. Total 6. Correct Option: **(B)**
14. Swap diagonal elements and change signs of off-diagonal elements: $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$. Correct Option: **(A)**
15. $|3A| = 3^3|A| = 27 \times 2 = 54$. Correct Option: **(C)**
16. Determinant of an odd-order skew-symmetric matrix is always 0. Correct Option: **(C)**
17. Multiplying A by itself results in the Identity matrix I. Correct Option: **(A)**
18. Solving the system: $x + y + z = 6$ and $x - y + z = 2$. Subtracting gives $2y = 4 \implies y = 2$. Correct Option: **(B)**
19. $|3AB| = 3^3|A||B| = 27 \times (-1) \times 3 = -81$. Correct Option: **(B)**
20. Using induction, for $A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$, the n-th power follows the rule for Jordan blocks: $\begin{bmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{bmatrix}$. Correct Option: **(A)**