CUET (UG) – MATHEMATICS

Chapter Test - Unit III: Calculus - Applications of Integrals

SOLUTIONS

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Solutions

1. **Correct Option: (C)**. Area $= \int_2^4 3\sqrt{x}dx = 3[\frac{2}{3}x^{3/2}]_2^4 = 2(4^{3/2} - 2^{3/2}) = 2(8 - 2\sqrt{2}) = 16 - 4\sqrt{2} = 4(4 - \sqrt{2})$.

2. **Correct Option: (A)**. Area = (Area of quadrant) - (Area of triangle) $= \frac{1}{4}\pi(1)^2 - \frac{1}{2}(1)(1) = \frac{\pi}{4} - \frac{1}{2}$.

3. **Correct Option: (B)**. Area $= 2\int_0^1 (x - x^2)dx = 2[\frac{x^2}{2} - \frac{x^3}{3}]_0^1 = 2(\frac{1}{2} - \frac{1}{3}) = 2(\frac{1}{6}) = 1/3$.

4. **Correct Option: (B)**. Intersection: $(2x)^2 = 4x \implies 4x^2 = 4x \implies x = 0, 1$. Area $= \int_0^1 (2\sqrt{x} - 2x)dx = [2 \cdot \frac{2}{3}x^{3/2} - x^2]_0^1 = \frac{4}{3} - 1 = 1/3$.

5. **Correct Option: (C)**. Total area πab. First quadrant is exactly one quarter: $\frac{\pi ab}{4}$.

6. **Correct Option: (A)**. Intersection: $x^2 + 3x^2 = 4 \implies x = 1$. Area $= \int_0^1 \sqrt{3}xdx + \int_1^2 \sqrt{4 - x^2}dx = \frac{\sqrt{3}}{2} + [\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}]_1^2 = \frac{\sqrt{3}}{2} + (0 + \pi) - (\frac{\sqrt{3}}{2} + \frac{\pi}{3}) = \frac{2\pi}{3}$. Wait, re-calc: $\pi - \pi/3 = 2\pi/3$? No, $16\pi/12 - 4\pi/12$... result is $\pi/3$.

7. **Correct Option: (A)**. Area $= \int_0^8 xdy = \int_0^8 y^{1/3}dy = [\frac{3}{4}y^{4/3}]_0^8 = \frac{3}{4}(16) = 12$.

8. **Correct Option: (A)**. Area $= 2\int_0^3 2\sqrt{x}dx = 4[\frac{2}{3}x^{3/2}]_0^3 = \frac{8}{3}(3\sqrt{3}) = 8\sqrt{3}$.

9. **Correct Option: (B)**. Intersection with y-axis: $2y - y^2 = 0 \implies y = 0, 2$. Area $= \int_0^2 (2y - y^2)dy = [y^2 - \frac{y^3}{3}]_0^2 = 4 - 8/3 = 4/3$.

10. **Correct Option: (A)**. Area $= \int_0^1 (e^x - e^{-x})dx = [e^x + e^{-x}]_0^1 = (e + 1/e) - (1 + 1) = e + 1/e - 2$.

11. **Correct Option: (A)**. Intersection $x = 0, 1$. Area $= \int_0^1 (\sqrt{x} - x^2)dx = [2/3 - 1/3] = 1/3$.

12. **Correct Option: (A)**. Area $= \int_0^{\pi/4} \sin 2xdx = [-\frac{1}{2}\cos 2x]_0^{\pi/4} = -\frac{1}{2}(0 - 1) = 1/2$.

13. **Correct Option: (B)**. $a = 2$, Latus rectum $x = 2$. Area $= 2\int_0^2 \sqrt{8x}dx = 4\sqrt{2}\int_0^2 x^{1/2}dx = 4\sqrt{2}[\frac{2}{3} \cdot 2\sqrt{2}] = \frac{32}{3}$.

14. **Correct Option: (B)**. $x = 2\sqrt{y}$. Area $= \int_2^4 2\sqrt{y}dy = 2[\frac{2}{3}y^{3/2}]_2^4 = \frac{4}{3}(8 - 2\sqrt{2})$.

15. **Correct Option: (A)**. Triangle with base from $x = 0$ to $x = 2$ (length 2) and height 1. Area $= 1/2 \times 2 \times 1 = 1$.

16. **Correct Option: (C)**. Intersection at $x = 2$. Area $= \int_0^2 \sqrt{6x}dx + \int_2^4 \sqrt{16 - x^2}dx = \frac{4\sqrt{12}}{3} + [\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\frac{x}{4}]_2^4 = \frac{8\sqrt{3}}{3} + (4\pi) - (\sqrt{12} + \frac{4\pi}{3}) = \frac{8\pi}{3} + \frac{4\sqrt{3}}{3}$.

17. **Correct Option: (A)**. Area $= \int_0^3 (x^2 + 2 - x)dx = [\frac{x^3}{3} + 2x - \frac{x^2}{2}]_0^3 = (9 + 6 - 9/2) = 15 - 4.5 = 10.5 = 21/2$.

18. **Correct Option: (B)**. This is the upper semi-circle of $x^2 + y^2 = a^2$. Area $= \frac{\pi a^2}{2}$.

19. **Correct Option: (D)**. Area $= \int_0^{\pi/4} \tan xdx = [\log|\sec x|]_0^{\pi/4} = \log\sqrt{2} = \frac{1}{2}\log 2$.

20. **Correct Option: (A)**. Intersection $x^2 = 4x \implies x = 0, 4$. Area $= \int_0^4 (2\sqrt{x} - x)dx = [2 \cdot \frac{2}{3}x^{3/2} - \frac{x^2}{2}]_0^4 = \frac{4}{3}(8) - 8 = \frac{32}{3} - \frac{24}{3} = 8/3$.