Chapter 7: Coordinate Geometry

This chapter introduces the language to describe the position and location of points on a plane using numbers. You will learn how to find distances, midpoints, and areas using these number pairs called coordinates.
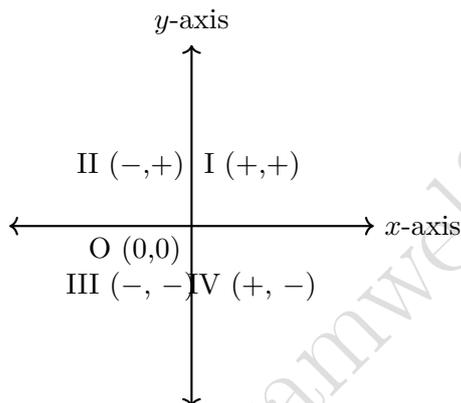
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1. The Cartesian Plane

The foundation of coordinate geometry is a plane formed by two perpendicular number lines.

1. **Cartesian Plane / Coordinate Plane**

$$\text{Plane} = x\text{-axis} \perp y\text{-axis}$$

- x-**axis**: The horizontal number line.
- y-**axis**: The vertical number line.
- **Origin** (O): The point of intersection $(0,0)$.
- **Quadrants**: The axes divide the plane into four regions (I, II, III, IV).

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2. Coordinates of a Point

Every point on the plane can be uniquely identified by an ordered pair of numbers.

2. **Coordinates of a Point** P

$$P(x, y)$$

- x-**coordinate (Abscissa)**: The perpendicular distance from the y-axis.
- y-**coordinate (Ordinate)**: The perpendicular distance from the x-axis.
- **Usage**: To plot a point $P(3, -2)$, move 3 units right (along x-axis) and 2 units down (parallel to y-axis).

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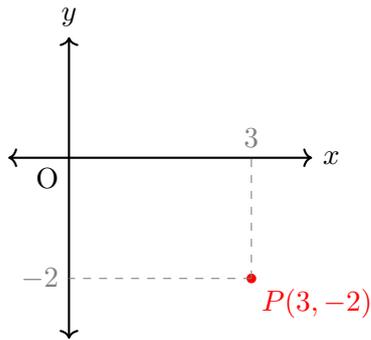
3. Distance Formula

Used to find the length of the line segment joining two points.

3. Distance between Two Points

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- $A(x_1, y_1)$: Coordinates of the first point.
- $B(x_2, y_2)$: Coordinates of the second point.
- AB: Length of the segment joining A and B.
- **Usage**: Directly apply the formula. Remember: Squaring the differences removes any negative sign issues.

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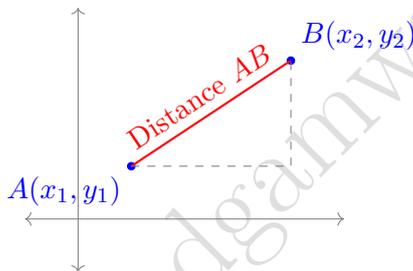
4. Section Formula

Used to find the coordinates of a point that divides a line segment in a given ratio.

4. Internal Division Point P

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \ \frac{my_2 + ny_1}{m + n} \right)$$

- $A(x_1, y_1), B(x_2, y_2)$: Endpoints of the segment.
- $m : n$: Ratio in which P divides AB internally (P lies between A and B).
- $P(x, y)$: Coordinates of the dividing point.

- **Usage**: Identify m and n correctly from the given ratio $m : n$. For the midpoint, use $m : n = 1 : 1$.

5. **Midpoint Formula (Special Case:** $m : n = 1 : 1$**)**

$$M(x, y) = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2} \right)$$

- M: Midpoint of segment AB.
- **Usage**: Shortcut to find the exact middle point of a segment.

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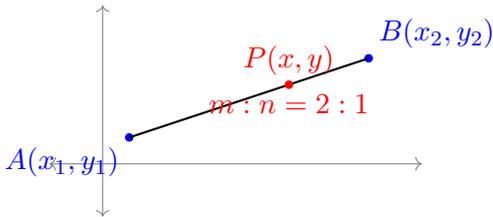
5. Area of a Triangle

Used to find the area when vertices' coordinates are known.

6. **Area using Coordinates**

$$\text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

- $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$: Vertices of triangle ABC.
- **Area**: The absolute value ensures a positive area.
- **Usage**: Write coordinates in order. The formula gives zero if points are collinear (a check for collinearity).

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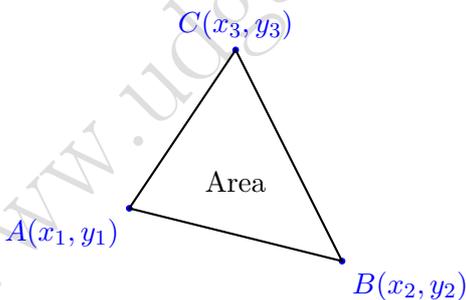
6. Collinearity of Points

Three points are collinear (lie on the same straight line) if the area of the triangle formed by them is zero.

7. **Condition for Collinearity**

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

- This is simply the area formula set equal to zero.
- **Usage**: To check if points A, B, C are in a straight line without plotting them.

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Quick Revision Summary

Here are all the essential formulas from this chapter for a quick recap.

1. **Distance Formula**
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. **Section Formula (Internal Division)**
$$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \ \frac{my_2 + ny_1}{m + n} \right)$$

3. **Midpoint Formula**
$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \ \frac{y_1 + y_2}{2} \right)$$

4. **Area of a Triangle**
$$\text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

5. **Condition for Collinearity**
$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$