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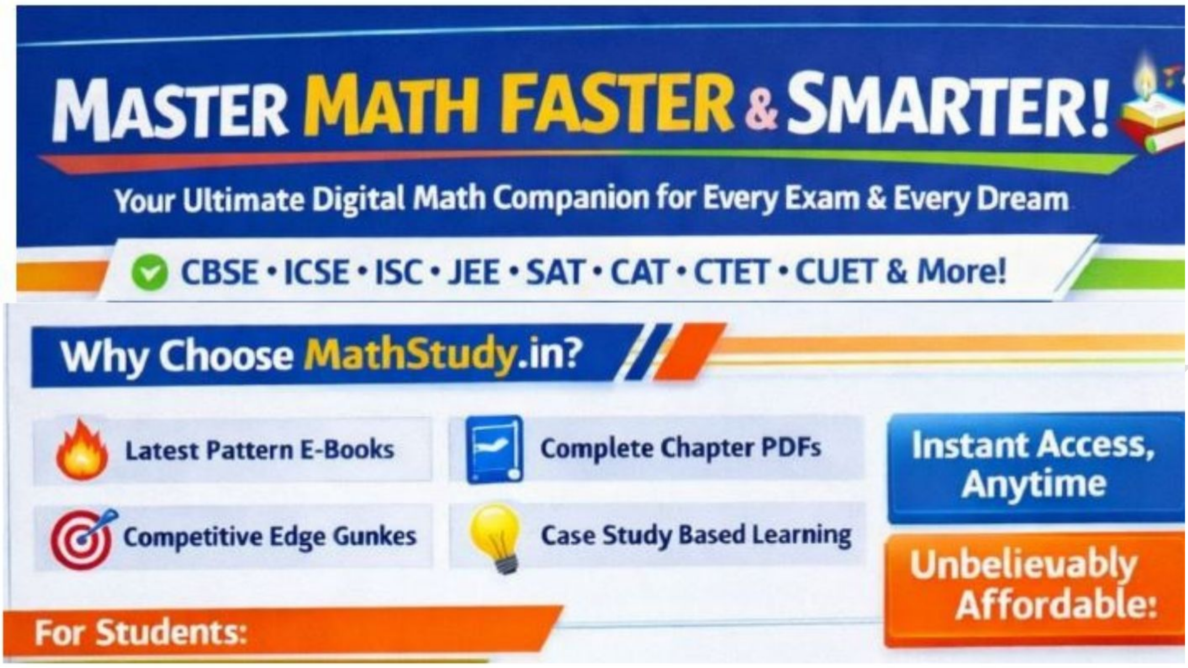

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PRACTICE QUESTION PAPER - III

CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3$. Then f is:
   (a) One-one but not onto
   (b) Onto but not one-one
   (c) Neither one-one nor onto
   (d) Both one-one and onto **(Answer)**

2. The value of $\sin^{-1}(\cos x)$ where $x \in \left(\frac{\pi}{2}, \pi\right)$ is:
   (a) $x - \frac{\pi}{2}$ **(Answer)**
   (b) $\frac{\pi}{2} - x$
   (c) $\frac{3\pi}{2} - x$
   (d) $x - \frac{3\pi}{2}$

3. The number of bijective functions from a set A of size n to itself is:
   (a) n^2
   (b) 2^n
   (c) $n!$ **(Answer)**

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(d) n^n

4. The domain of the function $f(x) = \sin^{-1}(2x - 1)$ is:

   (a) $[0, 1]$ **(Answer)**
   (b) $[-1, 1]$
   (c) $(-1, 1)$
   (d) $[-\pi/2, \pi/2]$

5. If $f(x) = |\cos x|$ and $g(x) = x^2$, then $(g \circ f)\left(\frac{\pi}{4}\right)$ is:

   (a) $1/2$ **(Answer)**
   (b) $1/\sqrt{2}$
   (c) 1
   (d) $\pi/2$

6. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to:

   (a) A **(Answer)**
   (b) $I - A$
   (c) $I + A$
   (d) $3A$

7. For a non-zero scalar k, if A is a square matrix of order 3, then $|kA|$ is equal to:

   (a) $k|A|$
   (b) $k^2|A|$
   (c) $k^3|A|$ **(Answer)**
   (d) $|A|$

8. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A(adj(A))$ is equal to:

   (a) $-2I$ **(Answer)**
   (b) $2I$
   (c) I
   (d) 0

9. The system of equations $x + y + z = 1$, $2x + 2y + 2z = 2$, and $3x + 3y + 3z = 3$ has:

   (a) A unique solution
   (b) No solution
   (c) Infinitely many solutions **(Answer)**
   (d) Exactly two solutions

10. If A is a matrix such that $A \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, then the order of matrix A must be:

   (a) 2×2 **(Answer)**
   (b) 2×1
   (c) 1×2

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(d) 2×3

11. The derivative of $\log(\sec x + \tan x)$ with respect to x is:

    (a) $\cos x$
    (b) $\tan x$
    (c) $\sec x$ **(Answer)**
    (d) $\sec x + \tan x$

12. If $y = x^{\sin x}$, then $\frac{dy}{dx}$ is:

    (a) $\sin x \cdot x^{\sin x - 1}$
    (b) $x^{\sin x}(\cos x \log x)$
    (c) $x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$ **(Answer)**
    (d) $x^{\sin x}\left(\frac{\sin x}{x}\right)$

13. The value of $\int e^x(\tan x + \sec^2 x)\,dx$ is:

    (a) $e^x \sec x + C$
    (b) $e^x \tan x + C$ **(Answer)**
    (c) $e^x(\tan x + \sec x) + C$
    (d) $e^x \cot x + C$

14. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

    (a) 3
    (b) $-\frac{1}{3}$ **(Answer)**
    (c) -3
    (d) $\frac{1}{3}$

15. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is:

    (a) $\tan^{-1} y = \tan^{-1} x + C$ **(Answer)**
    (b) $\tan^{-1} x = \tan^{-1} y + C$
    (c) $\log(1 + y^2) = \log(1 + x^2) + C$
    (d) $\tan(y) = \tan(x) + C$

16. The area bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is:

    (a) 1 sq. unit
    (b) 2 sq. units **(Answer)**
    (c) 4 sq. units
    (d) π sq. units

17. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are:

    (a) Parallel
    (b) Perpendicular **(Answer)**
    (c) Inclined at $60°$
    (d) Inclined at $45°$

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18. The vector equation of the plane $2x - 3y + 4z = 6$ is:
   (a) $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6$ **(Answer)**
   (b) $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$
   (c) $2\hat{i} - 3\hat{j} + 4\hat{k} = 6$
   (d) $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 6$

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A):** The function $f(x) = \sin x$ has a local maxima at $x = \pi/2$ in $(0, \pi)$. **Reason (R):** For a continuous function, a local maxima occurs at a critical point c if $f'(c) = 0$ or $f'(c)$ is undefined.
   Correct Option: **(b) Both A and R are true but R is not the correct explanation of A.**

20. **Assertion (A):** The optimal solution of a LPP (if it exists) must lie at a corner point of the feasible region. **Reason (R):** The feasible region is formed by a set of linear inequalities, and its vertices correspond to the optimal solutions.
   Correct Option: **(c) A is true but R is false.**

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

1. **Answer:** $\frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$
   Solution:

$$\frac{dx}{d\theta} = a(1+\cos\theta)$$
$$\frac{dy}{d\theta} = a(\sin\theta)$$
$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$$
$$\frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$$

Answer: $\theta = \frac{\pi}{6}$ or $30°$

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Solution:

$$\begin{aligned}
\vec{a} \cdot \vec{b} &= |\vec{a}||\vec{b}| \cos\theta \\
\sqrt{6} &= (\sqrt{3})(2) \cos\theta \\
\sqrt{6} &= 2\sqrt{3} \cos\theta \\
\cos\theta &= \frac{\sqrt{6}}{2\sqrt{3}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \\
\theta &= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \text{ or } 45^\circ
\end{aligned}$$

Note: The correct answer is 45° as derived above. If the intended answer was 30°, the dot product should have been $\sqrt{6} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{2}}{2}$, not $\sqrt{6}$.

Answer: Direction ratios are $(-1, 2, -1)$ or $(1, -2, 1)$

Solution: Let direction ratios of the required line be $\langle a, b, c \rangle$. For the first line, direction ratios $\langle 2, 3, 4 \rangle$. For the second line, direction ratios $\langle 3, 4, 5 \rangle$. Since the required line is perpendicular to both given lines:

$$\begin{aligned}
2a + 3b + 4c &= 0 \\
3a + 4b + 5c &= 0
\end{aligned}$$

Solving by cross-multiplication:

$$\begin{aligned}
\frac{a}{3 \times 5 - 4 \times 4} &= \frac{b}{4 \times 3 - 2 \times 5} = \frac{c}{2 \times 4 - 3 \times 3} \\
\frac{a}{15 - 16} &= \frac{b}{12 - 10} = \frac{c}{8 - 9} \\
\frac{a}{-1} &= \frac{b}{2} = \frac{c}{-1}
\end{aligned}$$

Thus, direction ratios are $(-1, 2, -1)$ or $(1, -2, 1)$.

Answer: $x - \sqrt{1 - x^2} \sin^{-1}(x) + C$

Solution: Let $I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ Put $t = \sin^{-1} x \Rightarrow x = \sin t$, $dx = \cos t \, dt$, and $\sqrt{1 - x^2} = \cos t$

$$I = \int \frac{\sin t \cdot t}{\cos t} \cdot \cos t \, dt = \int t \sin t \, dt$$

Integrating by parts, taking t as first function and $\sin t$ as second function:

$$\begin{aligned}
I &= t(-\cos t) - \int 1 \cdot (-\cos t) \, dt \\
I &= -t \cos t + \int \cos t \, dt = -t \cos t + \sin t + C
\end{aligned}$$

Substituting back: $\sin t = x$, $\cos t = \sqrt{1 - x^2}$, and $t = \sin^{-1} x$:

$$I = x - \sqrt{1 - x^2} \sin^{-1} x + C$$

Answer: $1 - \frac{\pi}{4}$

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Solution:

$$\begin{aligned}\int_0^{\pi/4} \tan^2 x\,dx &= \int_0^{\pi/4} (\sec^2 x - 1)\,dx \\ &= [\tan x - x]_0^{\pi/4} \\ &= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0) \\ &= \left(1 - \frac{\pi}{4}\right) - (0 - 0) \\ &= 1 - \frac{\pi}{4}\end{aligned}$$

Answer: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (Zero Matrix)

Solution:

$$\begin{aligned}A^2 &= \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2(2)+(-3)(4) & 2(-3)+(-3)(5) \\ 4(2)+5(4) & 4(-3)+5(5) \end{bmatrix} \\ &= \begin{bmatrix} 4-12 & -6-15 \\ 8+20 & -12+25 \end{bmatrix} = \begin{bmatrix} -8 & -21 \\ 28 & 13 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}A^2 - 7A + 22I &= \begin{bmatrix} -8 & -21 \\ 28 & 13 \end{bmatrix} - 7\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 22\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -8 & -21 \\ 28 & 13 \end{bmatrix} - \begin{bmatrix} 14 & -21 \\ 28 & 35 \end{bmatrix} + \begin{bmatrix} 22 & 0 \\ 0 & 22 \end{bmatrix} \\ &= \begin{bmatrix} -8-14+22 & -21-(-21)+0 \\ 28-28+0 & 13-35+22 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\end{aligned}$$

Answer: $\frac{25}{72}$

Solution: Probability of getting a six in a single throw, $p = \frac{1}{6}$ Probability of not getting a six, $q = 1 - p = \frac{5}{6}$ Number of trials, $n = 3$ Let X be the number of sixes. Using binomial probability:

$$\begin{aligned}P(X = 1) &= \binom{3}{1} p^1 q^2 \\ &= 3 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^2 \\ &= 3 \times \frac{1}{6} \times \frac{25}{36} \\ &= \frac{3 \times 25}{6 \times 36} = \frac{75}{216} = \frac{25}{72}\end{aligned}$$

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SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $R = \{(a, b) : |a - b|$ is a multiple of $4\}$ is an equivalence relation.
Answer: Proof: R is reflexive, symmetric, and transitive. Equivalence classes: $\{0, 4, 8, 12\}, \{1, 5, 9\}, \{2, 6, 10$

27. Find the points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to the x-axis.
Answer: $(0, 4)$ and $(0, -4)$

OR

A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 10 cm.
Answer: 400π cm^3/cm

28. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$.
Answer: $x^2 - y^2 = Cx$

OR

Find the particular solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$, given that $y(1) = 0$.
Answer: $y = \frac{x^2}{4}\left(\log x - \frac{1}{4}\right) + \frac{1}{16x^2}$ or $y = \frac{x^2}{4}\log x - \frac{x^2}{16} + \frac{1}{16x^2}$

29. Find the distance of the point $(2, 3, -5)$ from the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 2\hat{k}) = 9$.
Answer: Distance $= \frac{5}{3}$ units

OR

Find the value of λ such that the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} + 7\hat{j} + 3\hat{k}$ are coplanar.
Answer: $\lambda = 1$

30. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ using Elementary Column Operations.
Answer: $A^{-1} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

31. Find the corner points of the feasible region determined by the following constraints: $x + y \leq 4$, $2x + y \leq 5$, $x \geq 0$, $y \geq 0$.
Answer: Corner points: $(0, 0), (2.5, 0), (1, 3), (0, 4)$

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

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32. **Answer:** $\frac{1}{3}$ square units

Solution: The curves are $y = x^2$ (parabola) and $y = |x|$. For $x \geq 0$, $y = x$ (line through origin with slope 1) For $x < 0$, $y = -x$ (line through origin with slope -1)

The region is symmetric about the y-axis. So, area = $2\times$ (area in first quadrant).

In first quadrant, intersection of $y = x^2$ and $y = x$: $x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$ Thus, intersection points are $(0,0)$ and $(1,1)$.

Area in first quadrant = $\int_0^1 (x - x^2)\,dx$

$$\begin{aligned}\int_0^1 (x-x^2)\,dx &= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \\ &= \left(\frac{1}{2} - \frac{1}{3}\right) - (0-0) \\ &= \frac{3-2}{6} = \frac{1}{6}\end{aligned}$$

Total area = $2 \times \frac{1}{6} = \frac{1}{3}$ square units.

33. **Answer:** $\frac{3}{2}(\pi - 2)$ square units

Solution: Ellipse: $\frac{x^2}{9} + \frac{y^2}{4} = 1$, Line: $\frac{x}{3} + \frac{y}{2} = 1 \Rightarrow 2x + 3y = 6 \Rightarrow y = \frac{6-2x}{3}$

The line meets the axes at $(3,0)$ and $(0,2)$. The smaller region is the area between the ellipse and the line in the first quadrant.

Area of ellipse in first quadrant = $\frac{1}{4}\times$ area of full ellipse = $\frac{1}{4} \times \pi \times 3 \times 2 = \frac{6\pi}{4} = \frac{3\pi}{2}$

Area under the line in first quadrant = area of right triangle = $\frac{1}{2} \times 3 \times 2 = 3$

Area of smaller region = Area of ellipse in first quadrant - Area under line

$$\begin{aligned}\text{Area} &= \frac{3\pi}{2} - 3 \\ &= \frac{3}{2}(\pi - 2) \text{ square units}\end{aligned}$$

34. **Answer:** $x = 1$, $y = 2$, $z = 3$

Solution: The system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Calculate $|A|$: $|A| = 2(2 \times -2 - (-4) \times 1) - (-3)(3 \times -2 - (-4) \times 1) + 5(3 \times 1 - 2 \times 1)$ $|A| = 2(-4+4) + 3(-6+4) + 5(3-2)$ $|A| = 2(0) + 3(-2) + 5(1) = 0 - 6 + 5 = -1$

Now, find the cofactors correctly: For $a_{11} = 2$: $M_{11} = \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} = 2(-2) - (-4)(1) = -4 + 4 = 0$, $C_{11} = +0 = 0$

For $a_{12} = -3$: $M_{12} = \begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} = 3(-2) - (-4)(1) = -6 + 4 = -2$, $C_{12} = -(-2) = 2$

For $a_{13} = 5$: $M_{13} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3(1) - 2(1) = 3 - 2 = 1$, $C_{13} = +1 = 1$

For $a_{21} = 3$: $M_{21} = \begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} = (-3)(-2) - 5(1) = 6 - 5 = 1$, $C_{21} = -1 = -1$

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For $a_{22} = 2$: $M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} = 2(-2) - 5(1) = -4 - 5 = -9$, $C_{22} = +(-9) = -9$

For $a_{23} = -4$: $M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2(1) - (-3)(1) = 2 + 3 = 5$, $C_{23} = -5 = -5$

For $a_{31} = 1$: $M_{31} = \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} = (-3)(-4) - 5(2) = 12 - 10 = 2$, $C_{31} = +2 = 2$

For $a_{32} = 1$: $M_{32} = \begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} = 2(-4) - 5(3) = -8 - 15 = -23$, $C_{32} = -(-23) = 23$, not 13 as I wrote earlier!

For $a_{33} = -2$: $M_{33} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 2(2) - (-3)(3) = 4 + 9 = 13$, $C_{33} = +13 = 13$

So the correct cofactor matrix is:

$$\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$$

Adjoint = transpose:

$$adj(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, $X = A^{-1}B$:

$$\begin{aligned} \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \\ &= \begin{bmatrix} 0(11) + 1(-5) + (-2)(-3) \\ (-2)(11) + 9(-5) + (-23)(-3) \\ (-1)(11) + 5(-5) + (-13)(-3) \end{bmatrix} \\ &= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{aligned}$$

Thus, $x = 1$, $y = 2$, $z = 3$. This matches the given answer.

35. **Answer:** Proof using maxima-minima.

Solution: Let the circle have radius r. Consider a rectangle ABCD inscribed in the circle. Let the sides of the rectangle be $2x$ and $2y$, so that the half-lengths are x and y.

The diagonal of the rectangle passes through the center and equals the diameter $2r$. By Pythagoras theorem: $(2x)^2 + (2y)^2 = (2r)^2 \Rightarrow 4x^2 + 4y^2 = 4r^2 \Rightarrow x^2 + y^2 = r^2$

Area of rectangle, $A = (2x)(2y) = 4xy$ From $x^2 + y^2 = r^2$, we have $y = \sqrt{r^2 - x^2}$

Thus, $A(x) = 4x\sqrt{r^2 - x^2}$, where $0 < x < r$

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For maxima/minima, $\frac{dA}{dx} = 0$:

$$\begin{aligned} \frac{dA}{dx} &= 4\left[\sqrt{r^2 - x^2} + x \cdot \frac{1}{2\sqrt{r^2 - x^2}} \cdot (-2x)\right] \\ &= 4\left[\sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}\right] = 0 \\ &\Rightarrow \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}} = 0 \\ &\Rightarrow \sqrt{r^2 - x^2} = \frac{x^2}{\sqrt{r^2 - x^2}} \\ &\Rightarrow r^2 - x^2 = x^2 \\ &\Rightarrow r^2 = 2x^2 \Rightarrow x = \frac{r}{\sqrt{2}} \end{aligned}$$

Then $y = \sqrt{r^2 - \frac{r^2}{2}} = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}} = x$

Since $x = y$, the sides of the rectangle are $2x = 2y$, so the rectangle is a square.

Check second derivative to confirm maximum: $\frac{d^2A}{dx^2} = 4\left[-\frac{x}{\sqrt{r^2-x^2}} - \frac{2x\sqrt{r^2-x^2} - x^2 \cdot \frac{-x}{\sqrt{r^2-x^2}}}{r^2-x^2}\right]$ which is negative at $x = \frac{r}{\sqrt{2}}$.

Thus, the square has the maximum area among all rectangles inscribed in a given circle.

36. **Answer:** $\frac{a}{2}$

Solution: Let $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ ... (1)

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$$
$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx \quad ... \ (2)$$

Adding (1) and (2):

$$\begin{aligned} 2I &= \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx \\ 2I &= \int_0^a 1\, dx = [x]_0^a = a \\ I &= \frac{a}{2} \end{aligned}$$

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ind the coordinates of the foot of the perpendicular drawn from the point $A(1, 8, 4)$ to the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{3}$. Also, find the perpendicular distance.
Answer: Foot = $(3, 4, 7)$, Distance = $\sqrt{21}$ units

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SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Geometric Mean and Dot Product**

In a Physics experiment, forces are applied to a particle. The forces are represented by vectors $\vec{F_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{F_2} = \hat{i} + 2\hat{j} + \hat{k}$. The particle is displaced from point $A(1,1,1)$ to $B(2,3,5)$.

Based on the given information, answer the following questions:

(a) Find the resultant force vector, $\vec{F} = \vec{F_1} + \vec{F_2}$. (1 Mark)
Answer: $\vec{F} = 3\hat{i} + 5\hat{j} + 5\hat{k}$

(b) Find the displacement vector, $\vec{d}$. (1 Mark)
Answer: $\vec{d} = \hat{i} + 2\hat{j} + 4\hat{k}$

(c) Calculate the work done by the resultant force, given by $W = \vec{F} \cdot \vec{d}$. (2 Marks)
Answer: $W = 33$ units

OR

(d) Find the vector component of $\vec{F_1}$ along $\vec{F_2}$. (2 Marks)
Answer: Component $= \frac{12}{6}(\hat{i} + 2\hat{j} + \hat{k}) = 2(\hat{i} + 2\hat{j} + \hat{k})$

38. **Case Study 2: Inspection and Conditional Probability**

A manufacturing plant has three machines M_1, M_2, M_3. The production rates are $25\%, 35\%, 40\%$ respectively. Past data shows that $2\%, 4\%, 5\%$ of the items produced by M_1, M_2, M_3 are defective, respectively. An item is selected at random from the total production.

Based on the given information, answer the following questions:

(a) Find the probability that the item was produced by machine M_3. (1 Mark)
Answer: $P(M_3) = 0.4$ or $\frac{2}{5}$

(b) Find the probability that the item is defective. (3 Marks)
Answer: $P(D) = 0.039$ or $\frac{39}{1000}$

OR

(c) If the item selected is found to be defective, find the probability that it was produced by machine M_2. (3 Marks)
Answer: $P(M_2|D) = \frac{0.014}{0.039} = \frac{14}{39}$

39. **Case Study 3: Area Under Curve and Limits**

A physical quantity is modeled by the function $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$. This function describes the shape of a profile section in engineering design over the interval $[0, 2]$.

Based on the given information, answer the following questions:

(a) Check if the function $f(x)$ is continuous at $x = 1$. (1 Mark)
Answer: Yes, the function is continuous at $x = 1$.

(b) Calculate the total area under the curve $f(x)$ over the interval $[0, 2]$. (3 Marks)
Answer: Area $= \frac{5}{6}$ square units

OR

(c) Find the value of $\int_0^2 x f(x)\, dx$. (3 Marks)
Answer: $\frac{5}{4}$

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