PRACTICE QUESTION PAPER - IV
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. The relation $R = \{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ on set $A = \{1,2,3\}$ is:
   - (a) Only Reflexive
   - (b) Only Transitive
   - (c) Reflexive and Transitive
   - (d) An Equivalence relation
2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |x|$, then the range of f is:
   - (a) $\mathbb{R}$
   - (b) $[0, \infty)$
   - (c) $(-\infty, 0)$
   - (d) $(0, \infty)$
3. The value of $\cos\left(\sec^{-1} x + \csc^{-1} x\right)$ for $|x| \geq 1$ is:
   - (a) 1
   - (b) -1
   - (c) 0

(d) $\frac{\pi}{2}$

4. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ and $xy < 1$, then $x + y + xy$ is equal to:

   (a) 1
   (b) 0
   (c) -1
   (d) $\frac{1}{2}$

5. The set $A = \{a, b, c\}$ and $B = \{1, 2\}$. The number of onto functions from A to B is:

   (a) 6
   (b) 7
   (c) 2
   (d) 8

6. If the matrix $A = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$, then A^3 is equal to:

   (a) $\begin{bmatrix} 0 & a^3 \\ -a^3 & 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & -a^3 \\ a^3 & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} a^3 & 0 \\ 0 & a^3 \end{bmatrix}$
   (d) $\begin{bmatrix} -a^3 & 0 \\ 0 & -a^3 \end{bmatrix}$

7. If A is an invertible matrix and $A^{-1} = \begin{bmatrix} 3 & -1 \\ -4 & 1 \end{bmatrix}$, then the matrix A is:

   (a) $\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$
   (b) $\begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & -1 \\ -4 & 3 \end{bmatrix}$
   (d) $\begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

8. If A is a square matrix of order n, then $|adj(adj(A))|$ is:

   (a) $|A|^{n-1}$
   (b) $|A|^{n(n-1)}$
   (c) $|A|^{(n-1)^2}$
   (d) $|A|^{n^2}$

9. If A is a 3×3 matrix and $adj(A) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then $|A|$ is:

(a) 5
(b) 25
(c) 125
(d) -5

10. Let A be a non-zero column matrix. The rank of AA^T is:

(a) 0
(b) 1
(c) 2
(d) 3

11. If $y = \log(x + \sqrt{x^2 + a^2})$, then $\frac{dy}{dx}$ is:

(a) $\frac{1}{\sqrt{x^2+a^2}}$
(b) $\frac{1}{x+\sqrt{x^2+a^2}}$
(c) $\frac{x}{\sqrt{x^2+a^2}}$
(d) $\sqrt{x^2 + a^2}$

12. The particular solution of the differential equation $\frac{dy}{dx} = 2x$ when $y(0) = 0$ is:

(a) $y = x^2$
(b) $y = x^2 + 1$
(c) $y = 2x^2$
(d) $y = 2x^2 + 1$

13. The rate of change of area of a circle with respect to its diameter is:

(a) $2\pi r$
(b) πr
(c) π
(d) 2π

14. The maximum value of the function $f(x) = 3 - 2\sin x$ is:

(a) 5
(b) 3
(c) 1
(d) -1

15. The value of $\int \frac{1}{\sin^2 x \cos^2 x}\, dx$ is:

(a) $\tan x + \cot x + C$
(b) $\tan x - \cot x + C$
(c) $\sec x - \csc x + C$
(d) $-\tan x + \cot x + C$

16. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is:

(a) 1
(b) 2
(c) 3
(d) Not defined

17. The coordinates of the foot of the perpendicular from the origin on the plane $2x-3y+4z-6=0$ are:

(a) $\left(\frac{12}{29},-\frac{18}{29},\frac{24}{29}\right)$
(b) $(2,-3,4)$
(c) $(12,-18,24)$
(d) $\left(\frac{12}{13},-\frac{18}{13},\frac{24}{13}\right)$

18. If the scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is $|\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(a) 0
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{3}$

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A):** The function $f(x)=x^3-6x^2+15x-8$ is strictly increasing on $\mathbb{R}$. **Reason (R):** A differentiable function $f(x)$ is strictly increasing if $f'(x)>0$ for all x.

20. **Assertion (A):** If A and B are two matrices such that AB and BA are defined, then $AB=BA$. **Reason (R):** Matrix multiplication is generally commutative.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find the value of $\frac{dy}{dx}$ at $t=\frac{\pi}{2}$ if $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$.

22. Find the area of the triangle whose vertices are $A(1,1,1)$, $B(1,2,3)$, and $C(2,3,1)$.

OR

Show that the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-3}{-4}=\frac{y-2}{-3}=\frac{z-1}{-2}$ intersect.

23. Find $\int \sin^{-1}(\cos x)\, dx$.

OR

Find $\int \frac{dx}{\sqrt{7-6x-x^2}}$.

24. Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin x$ is neither one-one nor onto.

25. A die is tossed twice. Let E be the event "first toss shows a 5" and F be the event "sum of outcomes is greater than 9". Find $P(E|F)$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Simplify $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, where $x \neq 0$.

27. Find the maximum value of $4x + \frac{16}{x}$ for $x > 0$.

OR

Find the derivative of $\sin x$ w.r.t. x^2.

28. Find the general solution of the differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x$.

OR

Show that $y = Ax + B/x$ is a solution of the differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

29. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$.

OR

Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and passing through the point $(2, 2, 1)$.

30. Using properties of determinants, show that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$.

31. Minimize the objective function $Z = 5x + 10y$ subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, and $x, y \geq 0$. (Identify the corner points only).

SECTION D (20 Marks)

*This section comprises **4** questions of **5** marks each.*

33. Find the area of the region bounded by the curves $y^2 = 4x$ and $x^2 = 4y$.

OR

Find the area bounded by the curve $y = \sqrt{x}$ and the line $y = x$.

34. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find A^{-1}. Hence, solve the system of linear equations:

$$\begin{aligned} x - y + z &= 4 \\ x - 2y - 2z &= 9 \\ 2x + y + 3z &= 1 \end{aligned}$$

35. A window is in the form of a rectangle surmounted by a semicircle. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

Evaluate $\int \frac{x^2+1}{(x^2+4)(x^2+2)}\,dx$.

36. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

SECTION E (12 Marks)

*This section comprises **3** case study based questions of **4** marks each.*

37. **Case Study 1: Drone Trajectory and Collinearity**

A surveillance drone is flying in a path defined by a line segment AB connecting two points $A(1, 2, -1)$ and $B(4, 6, 8)$. A sensor is located at point $C(10, 14, 26)$.

Based on the given information, answer the following questions:

(a) Find the vector $\vec{AB}$. (1 Mark)

(b) Determine if the points A, B, and C are collinear. (1 Mark)

(c) Find the direction cosines of the line segment BC. (2 Marks)

OR

(d) Find the vector component of $\vec{AC}$ perpendicular to $\vec{AB}$. (2 Marks)

38. **Case Study 2: Life Insurance and Expected Value**

A life insurance company offers a policy where, for an annual premium of $Rs.500$, a person gets $Rs.10,000$ if they die within the year. The probability of a person dying in the age group covered by the policy is 0.005.

Let X be the random variable representing the gain (or loss) for the insurance company.

Based on the given information, answer the following questions:

(a) What is the value of X if the person survives the year? (1 Mark)

(b) Prepare the probability distribution of X. (1 Mark)

(c) Calculate the expected gain (or loss) $E(X)$ for the insurance company on one policy. (2 Marks)

OR

(d) Calculate the variance of X (You may use $E(X^2) = 50000$). (2 Marks)

39. **Case Study 3: Rate of Change and Volume**

A storage tank is in the shape of a cube. The side length of the cube is increasing at a constant rate of 5 cm/s due to expansion. The current side length is 10 cm.

Based on the given information, answer the following questions:

(a) Write the formula for the volume V of the cube in terms of its side a. (1 Mark)

(b) Find the rate at which the volume of the cube is increasing ($\frac{dV}{dt}$) when the side length is 10 cm. (3 Marks)

OR

(c) If the rate of change of volume were proportional to the square of the side, i.e., $\frac{dV}{dt} = ka^2$, find the value of the constant k. (3 Marks)