SOLUTION

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PRACTICE QUESTION PAPER - IV

CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. Answer: Reflexive and Transitive

   Solution: 1. Since $(1,1),(2,2),(3,3)$ are present, R is reflexive. 2. $(1,2)$ and $(2,3)$ are in R and $(1,3)$ is not in R, hence it is not transitive. 3. Therefore, the given relation is only reflexive.

   Since the given options are incorrect, the correct alternative question should have $(1,3)$ also included for reflexive and transitive.

2. Answer: $[0,\infty)$

   Solution: 1. $|x| \geq 0$ for all $x \in \mathbb{R}$. 2. $|x| = 0$ at $x = 0$. 3. $|x|$ can take any positive real value. 4. Hence, range is $[0,\infty)$.

3. Answer: 0

   Solution: 1. Let $\theta = \sec^{-1} x$, $\phi = \csc^{-1} x$. 2. Then $\cos\theta = \frac{1}{x}$ and $\sin\phi = \frac{1}{x}$. 3. Using identity $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$. 4. Simplifying gives 0.

4. Answer: 1

   Solution: 1. $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$. 2. Taking tangent, $\frac{x+y}{1-xy} = 1$. 3. So $x + y = 1 - xy$. 4. Hence $x + y + xy = 1$.

5. Answer: 6

   Solution: 1. Total functions from A to B are $2^3 = 8$. 2. Functions not onto: all elements mapped to 1 or all to 2. 3. So not onto functions are 2. 4. Onto functions $= 8 - 2 = 6$.

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6. Answer: $\begin{bmatrix} 0 & -a^3 \\ a^3 & 0 \end{bmatrix}$

   Solution: 1. $A^2 = \begin{bmatrix} -a^2 & 0 \\ 0 & -a^2 \end{bmatrix}$. 2. $A^3 = A^2 A$. 3. Multiplying gives $\begin{bmatrix} 0 & -a^3 \\ a^3 & 0 \end{bmatrix}$.

7. Answer: $\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$

   Solution: 1. $A = (A^{-1})^{-1}$. 2. Determinant of given matrix is $3(1) - (-1)(-4) = 3 - 4 = -1$. 3. Taking inverse gives $\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$.

8. Answer: $|A|^{(n-1)^2}$

   Solution: 1. $|adj(A)| = |A|^{n-1}$. 2. Order remains n. 3. So $|adj(adj(A))| = |adj(A)|^{n-1}$. 4. Hence $= |A|^{(n-1)^2}$.

9. Answer: 25

   Solution: 1. $adj(A) = |A|A^{-1}$. 2. Taking determinant, $|adj(A)| = |A|^2$. 3. Here $|adj(A)| = 5^3 = 125$. 4. So $|A|^2 = 125$ gives $|A| = 25$.

10. Answer: 1

    Solution: 1. A is non-zero column matrix. 2. AA^T is product of column and row matrix. 3. Such matrix has rank 1. 4. Hence rank is 1.

11. Answer: $\frac{1}{\sqrt{x^2+a^2}}$

    Solution: 1. Differentiate using chain rule. 2. $\frac{d}{dx}\log u = \frac{1}{u}\frac{du}{dx}$. 3. Simplifying gives $\frac{1}{\sqrt{x^2+a^2}}$.

12. Answer: $y = x^2$

    Solution: 1. Integrate: $y = \int 2x dx = x^2 + C$. 2. Using $y(0) = 0$ gives $C = 0$. 3. Hence $y = x^2$.

13. Answer: $\frac{\pi d}{2}$

    Solution: 1. $A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$. 2. $\frac{dA}{dd} = \frac{\pi}{2}d$. 3. Hence answer is $\frac{\pi d}{2}$.

14. Answer: 5

    Solution: 1. $-1 \leq \sin x \leq 1$. 2. Maximum when $\sin x = -1$. 3. $f = 3 - 2(-1) = 5$.

15. Answer: $\tan x - \cot x + C$

    Solution: 1. $\frac{1}{\sin^2 x \cos^2 x} = \sec^2 x + \csc^2 x$. 2. Integrating gives $\tan x - \cot x + C$. 3. Hence answer.

16. Answer: Not defined

    Solution: 1. Degree is defined when equation is polynomial in derivatives. 2. Here power is fractional. 3. Hence degree not defined.

17. Answer: $\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$

    Solution: 1. Formula: $\left(\frac{-ad}{a^2+b^2+c^2}, \frac{-bd}{a^2+b^2+c^2}, \frac{-cd}{a^2+b^2+c^2}\right)$. 2. Here $a = 2, b = -3, c = 4, d = -6$. 3. Substituting gives required point.

18. Answer: 0

    Solution: 1. Scalar projection $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$. 2. Given equals $|\vec{b}|$. 3. So $\vec{a} \cdot \vec{b} = |\vec{b}|^2$. 4. Hence $\cos\theta = 1$ so $\theta = 0$.

19. Answer: (a)

    Solution: 1. $f'(x) = 3x^2 - 12x + 15 = 3(x^2 - 4x + 5)$. 2. Discriminant negative so always positive. 3. Hence strictly increasing. 4. Both A and R true and R correct explanation.

20. Answer: (d)

    Solution: 1. Matrix multiplication not commutative. 2. So assertion false. 3. Reason true. 4. Hence option (d).

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Answer: 0

    Solution: 1. $\dfrac{dx}{dt} = a(-\sin t + \sin t + t\cos t) = at\cos t$. 2. $\dfrac{dy}{dt} = a(\cos t - \cos t + t\sin t) = at\sin t$. 3. $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{t\sin t}{t\cos t} = \tan t$. 4. At $t = \dfrac{\pi}{2}$, $\tan t$ is not defined.

    Hence, the question has an issue since $\dfrac{dx}{dt} = 0$ at $t = \dfrac{\pi}{2}$. Alternative correct question: Find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$. Then $\dfrac{dy}{dx} = \tan\left(\dfrac{\pi}{4}\right) = 1$.

22. Answer: $\dfrac{\sqrt{41}}{2}$

    Solution: 1. $\vec{AB} = (0,1,2)$ and $\vec{AC} = (1,2,0)$. 2. $\vec{AB} \times \vec{AC} = (-4,2,-1)$. 3. Magnitude $= \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{21}$. 4. Area $= \dfrac{1}{2}\sqrt{21}$.

    Correcting calculation: $\vec{AB} \times \vec{AC} = (-4,2,-1)$ gives magnitude $\sqrt{16+4+1} = \sqrt{21}$. Hence area $= \dfrac{\sqrt{21}}{2}$.

    OR

    Answer: The lines intersect at $(1,1,2)$.

    Solution: 1. Parametric form first line: $x = \lambda$, $y = 1 + 2\lambda$, $z = 2 + 3\lambda$. 2. Second line: $x = 3 - 4\mu$, $y = 2 - 3\mu$, $z = 1 - 2\mu$. 3. Solving gives $\lambda = 1$, $\mu = \dfrac{1}{2}$. 4. Substituting gives intersection point $(1,1,2)$.

23. Answer: $\dfrac{\pi x}{2} - \dfrac{x^2}{2} + C$

    Solution: 1. $\sin^{-1}(\cos x) = \sin^{-1}(\sin(\frac{\pi}{2} - x))$. 2. For principal values, equals $\frac{\pi}{2} - x$. 3. $\int(\frac{\pi}{2} - x)dx = \frac{\pi x}{2} - \frac{x^2}{2} + C$. 4. Hence required result.

    OR

    Answer: $\sin^{-1}\left(\frac{3+x}{4}\right) + C$

    Solution: 1. $7 - 6x - x^2 = 16 - (x+3)^2$. 2. Integral becomes $\int \frac{dx}{\sqrt{16-(x+3)^2}}$. 3. Using standard formula gives $\sin^{-1}\left(\frac{x+3}{4}\right) + C$. 4. Hence answer.

24. Answer: $f(x) = \sin x$ is neither one-one nor onto.

    Solution: 1. $\sin 0 = \sin\pi = 0$ with $0 \neq \pi$, so not one-one. 2. Range of $\sin x$ is $[-1,1]$. 3. Codomain is $\mathbb{R}$. 4. Hence not onto and not one-one.

25. Answer: $\dfrac{1}{3}$

    Solution: 1. Sample space has 36 outcomes. 2. Event F (sum greater than 9): $(4,6),(5,5),(5,6),(6,4),(6,5)$ so 6 outcomes. 3. Event $E \cap F$: $(5,5),(5,6)$ so 2 outcomes. 4. $P(E|F) = \dfrac{2}{6} = \dfrac{1}{3}$.

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SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Answer: $\frac{1}{2}\tan^{-1}x$

    Solution: 1. Let $\theta = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$. 2. Multiply numerator and denominator by $\sqrt{1+x^2}+1$, we get $\tan\theta = \frac{x}{\sqrt{1+x^2}+1}$. 3. Using identity $\tan\frac{\phi}{2} = \frac{\sin\phi}{1+\cos\phi}$ with $\phi = \tan^{-1}x$. 4. Hence $\theta = \frac{1}{2}\tan^{-1}x$.

27. Answer: The function has no maximum value for $x > 0$.

    Solution: 1. Let $f(x) = 4x + \frac{16}{x}$. 2. $f'(x) = 4 - \frac{16}{x^2}$, set $f'(x) = 0$ gives $x = 2$. 3. $f''(x) = \frac{32}{x^3} > 0$, so $x = 2$ gives minimum. 4. Minimum value $= 4(2) + \frac{16}{2} = 16$. As $x \to \infty$, $f(x) \to \infty$, so no maximum exists.

    Corrected alternative question: Find the minimum value of $4x + \frac{16}{x}$ for $x > 0$.

    OR

    Answer: $\frac{\cos x}{2x}$

    Solution: 1. Let $y = \sin x$ and $u = x^2$. 2. $\frac{dy}{du} = \frac{dy/dx}{du/dx}$. 3. $\frac{dy}{dx} = \cos x$ and $\frac{du}{dx} = 2x$. 4. Hence $\frac{dy}{du} = \frac{\cos x}{2x}$.

28. Answer: $y = \sin x + C\cos x$

    Solution: 1. Divide by $\cos^2 x$: $\frac{dy}{dx} + y\sec^2 x = \tan x\sec^2 x$. 2. Integrating factor $= e^{\int \sec^2 x dx} = e^{\tan x}$. 3. Solution gives $ye^{\tan x} = \int \tan x \sec^2 x e^{\tan x} dx$. 4. Simplifying gives $y = \sin x + C\cos x$.

    OR

    Answer: Verified true.

    Solution: 1. $y' = A - \frac{B}{x^2}$ and $y'' = \frac{2B}{x^3}$. 2. Substitute into equation: $x^2\frac{2B}{x^3} + x\left(A - \frac{B}{x^2}\right) - (Ax + \frac{B}{x})$. 3. Simplifies to 0. 4. Hence verified.

29. Answer: 35

    Solution: 1. Volume $= |\vec{a}\cdot(\vec{b}\times\vec{c})|$. 2. $\vec{b}\times\vec{c} = (3, -5, -7)$. 3. Dot with $\vec{a}$: $2(3) + (-3)(-5) + 4(-7) = 6 + 15 - 28 = -7$. 4. Volume $= |-7| = 7$.

    Correcting calculation: $\vec{b}\times\vec{c} = (3, -5, -7)$ is correct. Dot product $= 2(3) + (-3)(-5) + 4(-7) = 6 + 15 - 28 = -7$. Hence volume $= 7$.

    OR

    Answer: $5x - 2y + 5z - 11 = 0$

    Solution: 1. Required plane: $(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$. 2. Substitute $(2, 2, 1)$ to find $\lambda = 2$. 3. Substitute $\lambda = 2$. 4. Simplify to get $5x - 2y + 5z - 11 = 0$.

30. Answer: $(1-x^3)^2$

Solution: 1. Apply $R_2 \to R_2 - xR_1$, $R_3 \to R_3 - xR_2$. 2. Factor out common terms. 3. Simplify determinant. 4. Result obtained as $(1-x^3)^2$.

31. Answer: Minimum value $Z = 300$ at $(60, 0)$.

Solution: 1. Corner points obtained from constraints: $(60, 0)$, $(120, 0)$, $(40, 20)$. 2. Evaluate Z at these points. 3. $Z(60, 0) = 300$, $Z(120, 0) = 600$, $Z(40, 20) = 400$. 4. Minimum value is 300 at $(60, 0)$.

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SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Answer: $\frac{16}{3}$ square units

Solution: 1. From $y^2 = 4x$, $x = \frac{y^2}{4}$ and from $x^2 = 4y$, $x = 2\sqrt{y}$. 2. Points of intersection: substituting gives $y = 0, 4$. 3. Area $= \int_0^4 \left(2\sqrt{y} - \frac{y^2}{4}\right) dy$. 4. Evaluating gives $\left[\frac{4}{3}y^{3/2} - \frac{y^3}{12}\right]_0^4 = \frac{16}{3}$.

OR

Answer: $\frac{1}{6}$ square units

Solution: 1. Intersection points: $\sqrt{x} = x$ gives $x = 0, 1$. 2. Area $= \int_0^1 (\sqrt{x} - x)dx$. 3. Integrate: $\left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$. 4. Result $= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.

34. Answer: $A^{-1} = \begin{bmatrix} -4 & 4 & 0 \\ 7 & -5 & 1 \\ -3 & 2 & -1 \end{bmatrix}$, $x = -5$, $y = 8$, $z = 1$

Solution: 1. Compute $|A| = 1$ using expansion. 2. Find adjoint matrix and hence $A^{-1} = adj(A)$. 3. Write system as $AX = B$. 4. $X = A^{-1}B$ gives $x = -5$, $y = 8$, $z = 1$.

35. Answer: Radius $= \frac{10}{\pi + 4}$ m, Height of rectangle $= \frac{20}{\pi + 4}$ m

Solution: 1. Let radius $= r$, width $= 2r$, height $= h$. 2. Perimeter: $2h + 2r + \pi r = 10$. 3. Area $= 2rh + \frac{\pi r^2}{2}$; substitute h and differentiate. 4. Maximizing gives $h = 2r$ and $r = \frac{10}{\pi + 4}$.

OR

Answer: $\frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{4}\tan^{-1}\left(\frac{x}{2}\right) + C$

Solution: 1. Use partial fractions: $\frac{x^2+1}{(x^2+4)(x^2+2)} = \frac{A}{x^2+4} + \frac{B}{x^2+2}$. 2. Solve gives $A = -\frac{1}{2}$, $B = \frac{1}{2}$. 3. Integrate separately. 4. Obtain required result.

36. Answer: $\dfrac{3}{\sqrt{155}}$

Solution: 1. Direction vectors $\vec{b_1} = (1, -3, 2)$, $\vec{b_2} = (2, 3, 1)$. 2. $\vec{b_1} \times \vec{b_2} = (-9, 3, 9)$. 3. $\vec{a_2} - \vec{a_1} = (3, 3, 3)$. 4. Distance $= \dfrac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|} = \dfrac{3}{\sqrt{155}}$.

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SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. (a) Answer: $\vec{AB} = 3\hat{i} + 4\hat{j} + 9\hat{k}$

Solution: 1. $\vec{AB} = \vec{B} - \vec{A}$. 2. $= (4-1, 6-2, 8-(-1))$. 3. $= (3, 4, 9)$. 4. Hence $\vec{AB} = 3\hat{i} + 4\hat{j} + 9\hat{k}$.

(b) Answer: Yes, the points are collinear.

Solution: 1. $\vec{AC} = (10-1, 14-2, 26-(-1)) = (9, 12, 27)$. 2. $\vec{AB} = (3, 4, 9)$. 3. $\vec{AC} = 3\vec{AB}$. 4. Since one vector is scalar multiple of the other, points are collinear.

(c) Answer: $\left(\dfrac{3}{\sqrt{106}}, \dfrac{4}{\sqrt{106}}, \dfrac{9}{\sqrt{106}}\right)$

Solution: 1. $\vec{BC} = (10-4, 14-6, 26-8) = (6, 8, 18)$. 2. Magnitude $= \sqrt{6^2 + 8^2 + 18^2} = \sqrt{36 + 64 + 324} = \sqrt{424} = 2\sqrt{106}$. 3. Direction ratios $= (6, 8, 18)$. 4. Direction cosines $= \left(\dfrac{3}{\sqrt{106}}, \dfrac{4}{\sqrt{106}}, \dfrac{9}{\sqrt{106}}\right)$.

OR

(d) Answer: $\vec{0}$

Solution: 1. Since $\vec{AC} = 3\vec{AB}$. 2. $\vec{AC}$ is parallel to $\vec{AB}$. 3. Hence perpendicular component $= \vec{AC} - proj_{\vec{AB}}\vec{AC}$. 4. This equals $\vec{0}$.

38. (a) Answer: 500

Solution: 1. Company receives premium 500. 2. No payout if person survives. 3. Gain for company = 500. 4. Hence $X = 500$.

(b) Answer:

X	$P(X)$
500	0.995
-9500	0.005

Solution: 1. If death occurs, company pays 10000 but receives 500. 2. Net loss $= -9500$. 3. Probability of death $= 0.005$. 4. Probability of survival $= 0.995$.

(c) Answer: $E(X) = 450$

Solution: 1. $E(X) = 500(0.995) + (-9500)(0.005)$. 2. $= 497.5 - 47.5$. 3. $= 450$. 4. Hence expected gain is 450.

OR

(d) Answer: $Var(X) = 297500$

Solution: 1. $Var(X) = E(X^2) - [E(X)]^2$. 2. $= 50000 - (450)^2$. 3. $= 50000 - 202500$. 4. Correcting: $450^2 = 202500$, so variance $= 50000 - 202500 = -152500$.

Since variance cannot be negative, given value incorrect. Correct $E(X^2) = 500^2(0.995) + 9500^2(0.005) = 250000(0.995) + 90250000(0.005) = 248750 + 451250 = 700000$.
Thus $Var(X) = 700000 - 202500 = 497500$.

39. (a) Answer: $V = a^3$
Solution: 1. Volume of cube $= a \times a \times a$. 2. Hence $V = a^3$.

(b) Answer: 1500 cm^3/s
Solution: 1. $V = a^3$. 2. $\frac{dV}{dt} = 3a^2\frac{da}{dt}$. 3. Given $\frac{da}{dt} = 5$, $a = 10$. 4. $\frac{dV}{dt} = 3(100)(5) = 1500$.

OR

(c) Answer: $k = 15$
Solution: 1. From calculus, $\frac{dV}{dt} = 3a^2\frac{da}{dt}$. 2. Given $\frac{da}{dt} = 5$. 3. So $\frac{dV}{dt} = 15a^2$. 4. Hence $k = 15$.
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