PRACTICE QUESTION PAPER - V
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x}{1+|x|}$. Then f is:

   (a) One-one but not onto
   (b) Onto but not one-one
   (c) Both one-one and onto
   (d) Neither one-one nor onto

2. The number of relations on the set $\{a, b\}$ that contain (a, b) and (b, a) and are reflexive is:

   (a) 2
   (b) 4
   (c) 6
   (d) 8

3. The value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ is:

   (a) $\frac{7\pi}{6}$
   (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{6}$

(d) $-\frac{\pi}{6}$

4. The domain of $f(x) = \tan^{-1}\left(\sqrt{x^2-1}\right)$ is:

   (a) $(-\infty, -1] \cup [1, \infty)$

   (b) $[-1, 1]$

   (c) $(-\infty, \infty)$

   (d) $(-\infty, -1) \cup (1, \infty)$

5. If $f(x) = [x]$ (greatest integer function) and $g(x) = |x|$, then $f \circ g\left(-\frac{5}{3}\right)$ is:

   (a) -2

   (b) -1

   (c) 1

   (d) 2

6. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then the value of α is:

   (a) ± 1

   (b) ± 3

   (c) ± 5

   (d) ± 7

7. If A is a square matrix, then $A - A^T$ is a:

   (a) Symmetric matrix

   (b) Skew-symmetric matrix

   (c) Identity matrix

   (d) Diagonal matrix

8. The maximum number of non-zero entries in a 3×3 upper triangular matrix is:

   (a) 3

   (b) 4

   (c) 6

   (d) 9

9. Let A be a non-singular matrix of order 2. If $A = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$, then $(A^T)^{-1}$ is equal to:

   (a) $\begin{bmatrix} -1/2 & 1 \\ 3/2 & -2 \end{bmatrix}$

   (b) $\begin{bmatrix} -1/2 & 3/2 \\ 1 & -2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & -1/2 \\ -2 & 3/2 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

10. If $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$, then x equals:

    (a) -1
    (b) 2
    (c) -2
    (d) 1

11. The value of $\int_0^{\pi/2} \log(\cot x)\, dx$ is:

    (a) $\frac{\pi}{2}$
    (b) $\frac{\pi}{4}$
    (c) 0
    (d) $\log 2$

12. If $\log_e(xy) = x^2 + y^2$, then $\frac{dy}{dx}$ is:

    (a) $\frac{y(2x^2-1)}{x(1-2y^2)}$
    (b) $\frac{y(1-2x^2)}{x(2y^2-1)}$
    (c) $\frac{x(1-2y^2)}{y(2x^2-1)}$
    (d) $\frac{2x^2-1}{1-2y^2}$

13. The integrating factor of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$ is:

    (a) $\sin x$
    (b) $\csc x$
    (c) $\log(\sin x)$
    (d) $\tan x$

14. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is:

    (a) $\frac{6}{7}$
    (b) $\frac{2}{3}$
    (c) $\frac{7}{6}$
    (d) $\frac{2}{7}$

15. The intervals in which the function $f(x) = 4\sin x$ is strictly increasing in $(0, 2\pi)$ are:

    (a) $(0, \frac{\pi}{2})$
    (b) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
    (c) $(0, \frac{\pi}{2}) \cup \left(\frac{3\pi}{2}, 2\pi\right)$
    (d) $(\pi, 2\pi)$

16. The general solution of the differential equation $\frac{dy}{dx} = 2^{y-x}$ is:

(a) $2^{-y} = 2^{-x} + C$
(b) $2^{-y} = \log_2 e \cdot 2^{-x} + C$
(c) $-2^{-y} = \log_2 e \cdot 2^{-x} + C$
(d) $-2^{-y} = \frac{2^{-x}}{\log 2} + C$

17. The shortest distance between the x-axis and the y-axis is:

(a) 0
(b) 1
(c) 2
(d) ∞

18. The cosine of the angle between any two diagonals of a cube is:

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$
(d) 0

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A):** The vector $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ is perpendicular to $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$. **Reason (R):** Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a} \cdot \vec{b} = 0$.

20. **Assertion (A):** If A and B are two events, then $P(A \cap B) \leq P(A)$. **Reason (R):** The event $A \cap B$ is a subset of the event A.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$, where $-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$.

22. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, find the value of λ.

OR

Find the unit vector perpendicular to both $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$.

23. Show that $y = ce^{-\frac{1}{x}}$ is a solution of the differential equation $x^2\frac{dy}{dx} = y$.

OR

Find the derivative of $\cos^{-1}(2x^2 - 1)$ with respect to $\sqrt{1 - x^2}$.

24. Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$.

25. A family has two children. What is the probability that both are boys, given that at least one of them is a boy?

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$.

27. Evaluate $\int \frac{x^2-1}{(x-1)^2(x+2)}\, dx$.

OR

Evaluate $\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\, dx$.

28. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(1, 2)$.

OR

Find the general solution of the differential equation $(x^2 + xy)\, dy = x^2\, dx$.

29. Find the shortest distance between the lines $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ and $\frac{x-2}{3} = \frac{y-1}{0} = \frac{z-1}{2}$.

OR

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

30. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, verify that $A \cdot adj(A) = |A|I$.

31. A dealer wishes to purchase two types of items, A and B. He has Rs 15,000 to spend. The cost of item A is Rs 300 and item B is Rs 400. He can store at most 40 items. Formulate this as a Linear Programming Problem to maximize his profit if he earns a profit of Rs 50 and Rs 60 on items A and B respectively.

SECTION D (20 Marks)

*This section comprises **4** questions of **5** marks each.*

33. Using the method of integration, find the area of the region bounded by the lines $2x+y=4$, $3x-2y=6$, and $x-3y+5=0$.

OR

Find the area bounded by the curve $y=2x-x^2$ and the line $y=x$.

34. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB. Hence, solve the system of equations $x-y=3$, $2x+3y+4z=17$, $y+2z=7$.

35. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of the radius of the cone.

OR

Find $\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}} e^x \, dx$.

36. Find the equation of the plane that contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$, and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}+6\hat{k})+8=0$.

SECTION E (12 Marks)

*This section comprises **3** case study based questions of **4** marks each.*

37. **Case Study 1: Cost, Revenue, and Profit**

A furniture manufacturer produces chairs. The cost of producing x chairs is given by $C(x) = 400 + 5x + 0.01x^2$ and the price per chair is $p(x) = 10 - 0.005x$. The Revenue function is $R(x) = x \cdot p(x)$, and Profit is $P(x) = R(x) - C(x)$.

Based on the given information, answer the following questions:

(a) Find the Revenue function $R(x)$. (1 Mark)

(b) Find the Marginal Profit when $x = 20$ units are produced. (2 Marks)

OR

(c) Find the number of chairs x for which the total profit is maximum. (2 Marks)

38. **Case Study 2: Binomial Distribution and Quality Check**

A machine produces 10% defective bulbs. A quality control officer selects a sample of 5 bulbs for testing. The random variable X denotes the number of defective bulbs in the sample.

Based on the given information, answer the following questions:

(a) What is the probability that exactly 2 bulbs are defective? (1 Mark)

(b) Find the probability that at most 1 bulb is defective. (3 Marks)

OR

(c) Find the mean of the number of defective bulbs in the sample. (3 Marks)

39. **Case Study 3: Ship Navigation and Plane Geometry**

A ship is sailing in the sea. Its position is tracked relative to three coastal radar stations A, B, and C. The stations lie on a plane defined by $x + 2y - 2z = 5$. The ship is currently at point $S(3, 4, 1)$.

Based on the given information, answer the following questions:

(a) Find the vector normal to the plane. (1 Mark)

(b) Calculate the shortest distance of the ship S from the plane defined by the radar stations. (3 Marks)

OR

(c) Find the equation of the plane parallel to the given plane and passing through the origin. (3 Marks)