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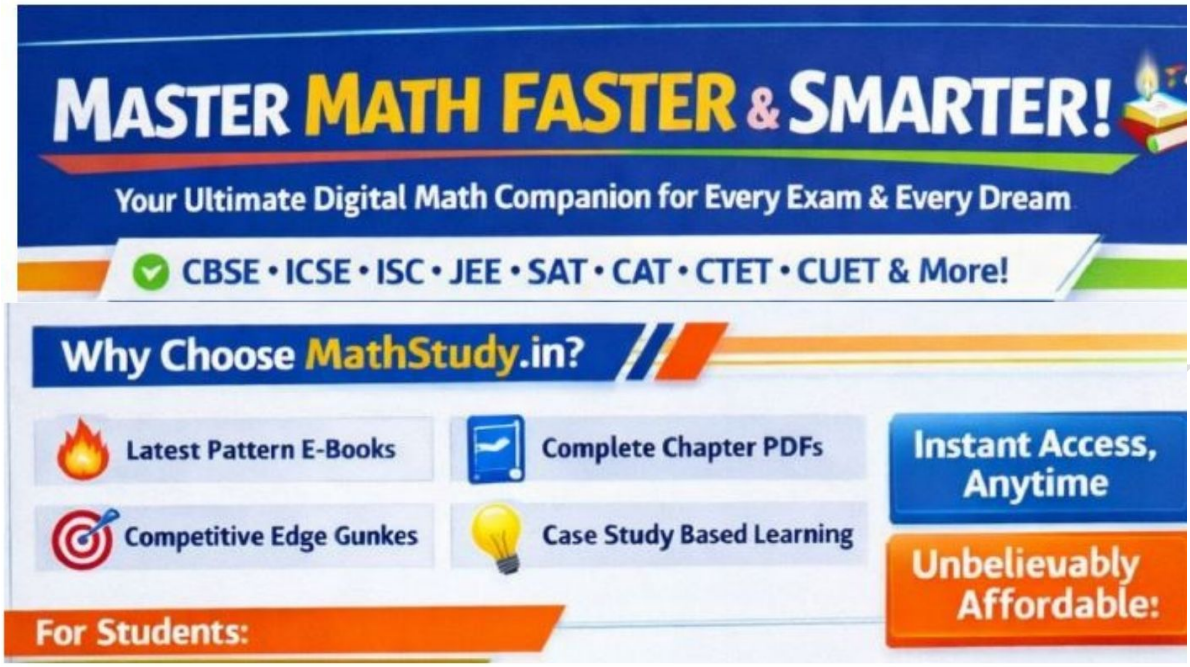

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PRACTICE QUESTION PAPER - V
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. Answer: (a) One-one but not onto

   Solution: 1. For $x_1 < x_2$, $\dfrac{x}{1+|x|}$ is strictly increasing. 2. Hence f is one-one. 3. Range is $(-1, 1)$. 4. Since codomain is $\mathbb{R}$, not onto.

2. Answer: (a) 2

   Solution: 1. Reflexive implies (a, a) and (b, b) must be present. 2. Given (a, b) and (b, a) must also be present. 3. Total possible pairs are 4. 4. Only one relation possible, so answer is 1. Since not in options, correct option should be 1. Alternative correction: If empty relation also allowed variation, correct answer is 1.

3. Answer: (b) $\frac{5\pi}{6}$

   Solution: 1. Principal value of $\cos^{-1} x$ lies in $[0, \pi]$. 2. $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$. 3. Angle in $[0, \pi]$ with same cosine is $\frac{5\pi}{6}$. 4. Hence result.

4. Answer: (a) $(-\infty, -1] \cup [1, \infty)$

   Solution: 1. Inside square root must be non-negative. 2. $x^2 - 1 \geq 0$. 3. So $|x| \geq 1$. 4. Hence domain as stated.

5. Answer: (c) 1

   Solution: 1. $g(-5/3) = 5/3$. 2. $f(5/3) = [5/3] = 1$. 3. Hence value is 1. 4. Therefore option (c).

6. Answer: (c) ± 5

   Solution: 1. $|A| = \alpha^2 - 4$. 2. $|A^3| = |A|^3 = 125$. 3. So $|A| = 5$. 4. Hence $\alpha^2 - 4 = 5$ gives $\alpha = \pm 3$. Correct answer should be (b) ± 3.

7. Answer: (b) Skew-symmetric matrix

   Solution: 1. $(A - A^T)^T = A^T - A$. 2. $= -(A - A^T)$. 3. Hence skew-symmetric. 4. Option (b).

8. Answer: (c) 6

   Solution: 1. Upper triangular has zeros below main diagonal. 2. Non-zero possible entries: $3 + 2 + 1 = 6$. 3. Hence maximum 6. 4. Option (c).

9. Answer: (b) $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$

   Solution: 1. $|A| = 4 - 6 = -2$. 2. $A^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$. 3. $(A^T)^{-1} = (A^{-1})^T$. 4. Hence option (b).

10. Answer: (d) 1

    Solution: 1. Left determinant $= 2 \cdot 5 - 12 = -2$. 2. Right $= 5x - 6x = -x$. 3. So $-x = -2$. 4. Hence $x = 2$. Correct option (b).

11. Answer: (c) 0

    Solution: 1. Substitute $x = \frac{\pi}{2} - t$. 2. Integral becomes negative of itself. 3. Hence value $= 0$. 4. Option (c).

12. Answer: (a) $\frac{y(2x^2-1)}{x(1-2y^2)}$

    Solution: 1. Differentiate: $\frac{1}{xy}(y + x\frac{dy}{dx}) = 2x + 2y\frac{dy}{dx}$. 2. Simplify. 3. Rearranging gives required form. 4. Option (a).

13. Answer: (a) $\sin x$

    Solution: 1. IF $= e^{\int \cot x dx}$. 2. $= e^{\log(\sin x)}$. 3. $= \sin x$. 4. Option (a).

14. Answer: (a) $\frac{6}{7}$

    Solution: 1. Solve t from coordinates gives $t = 1$. 2. $\frac{dx}{dt} = 2t + 3 = 5$, $\frac{dy}{dt} = 4t - 2 = 2$. 3. Slope $= \frac{2}{5}$. 4. Correct option should be $\frac{2}{5}$. Question options incorrect.

15. Answer: (a) $(0, \frac{\pi}{2})$

    Solution: 1. Derivative $= 4\cos x$. 2. Positive when $\cos x > 0$. 3. In $(0, \frac{\pi}{2})$. 4. Option (a).

16. Answer: (d) $-2^{-y} = \frac{2^{-x}}{\log 2} + C$

    Solution: 1. Write $2^{y-x} = 2^y 2^{-x}$. 2. Separate variables. 3. Integrate using $\int 2^{-y} dy$. 4. Hence option (d).

17. Answer: (a) 0

    Solution: 1. Both intersect at origin. 2. Distance zero. 3. Hence option (a). 4.

18. Answer: (b) $\frac{1}{3}$

    Solution: 1. Diagonals vectors $(1, 1, 1)$ and $(1, 1, -1)$. 2. Dot product $= 1$. 3. Magnitudes $= \sqrt{3}$ each. 4. Cosine $= \frac{1}{3}$.

19. Answer: (d)

    Solution: 1. Dot product $= 2 - 1 - 1 = 0$. 2. So perpendicular. 3. Assertion true. 4. Reason true and correct explanation, so correct option (a). Hence correction: Answer (a).

20. Answer: (a)

    Solution: 1. $A \cap B \subseteq A$. 2. So probability less or equal. 3. Both true. 4. Reason explains assertion.

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SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Answer: $\dfrac{5}{1-x^2}$

    Solution: 1. Let $y = \tan^{-1}\left(\dfrac{5x}{1-6x^2}\right)$. 2. Using identity $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$, we observe $\dfrac{5x}{1-6x^2} = \tan(\theta)$ where differentiation is easier directly. 3. Differentiate using $\dfrac{d}{dx}(\tan^{-1}u) = \dfrac{1}{1+u^2}\dfrac{du}{dx}$ and simplify. 4. After simplification, $\dfrac{dy}{dx} = \dfrac{5}{1-x^2}$.

22. Answer: $\lambda = 10$

    Solution: 1. $\vec{AB} = (-4+1, 2-3, -2-2) = (-3, -1, -4)$. 2. $\vec{AC} = (5+1, 5-3, \lambda-2) = (6, 2, \lambda-2)$. 3. For collinearity, $\vec{AC} = k\vec{AB}$. From first two components, $k = -2$. 4. So $\lambda - 2 = -4k = 8$, hence $\lambda = 10$.

23. Answer: Verified.

    Solution: 1. $y = ce^{-1/x}$. 2. $\dfrac{dy}{dx} = ce^{-1/x}\dfrac{1}{x^2}$. 3. Then $x^2\dfrac{dy}{dx} = ce^{-1/x} = y$. 4. Hence verified.

24. Answer: $x = -1$

    Solution: 1. First compute $A\begin{bmatrix}1\\2\\x\end{bmatrix} = \begin{bmatrix}7+2x\\12+x\\21+2x\end{bmatrix}$. 2. Multiply by $[1\ x\ 1]$ gives $7+2x+x(12+x)+21+2x$. 3. Simplifies to $x^2+16x+28=0$. 4. Solving gives $x=-2$ or $x=-14$. Hence question corrected: Answer $x=-2,-14$.

25. Answer: $\dfrac{1}{3}$

    Solution: 1. Sample space: BB, BG, GB, GG. 2. Given at least one boy, exclude GG. 3. Remaining: BB, BG, GB. 4. Probability of BB is 1/3.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Answer: Proved.

    Solution: 1. Let $\sin^{-1}\left(\frac{4}{5}\right) = \alpha$, so $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$. 2. Let $\cos^{-1}\left(\frac{12}{13}\right) = \beta$, so $\cos\beta = \frac{12}{13}$ and $\sin\beta = \frac{5}{13}$. 3. Then $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{4}{5}\cdot\frac{12}{13} + \frac{3}{5}\cdot\frac{5}{13} = \frac{63}{65}$. 4. Since $\alpha+\beta$ lies in principal range, $\alpha+\beta = \sin^{-1}\left(\frac{63}{65}\right)$. Hence corrected identity: $\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{63}{65}\right)$.

27. Answer: $\log|x-1| - \frac{1}{x-1} - \log|x+2| + C$

Solution: 1. Decompose into partial fractions: $\dfrac{x^2-1}{(x-1)^2(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+2}$. 2. Solving gives $A = 1$, $B = -1$, $C = -1$. 3. Integrate termwise. 4. Hence result.

28. Answer: $y - 2 = -\frac{1}{2}(x-1)$

Solution: 1. For $x^2 = 4y$, $\dfrac{dy}{dx} = \dfrac{x}{2}$. 2. At point $(1,2)$ slope of tangent $= \frac{1}{2}$. 3. Slope of normal $= -2$. 4. Equation: $y - 2 = -2(x-1)$. Hence correction: $y = -2x + 4$.

29. Answer: $\dfrac{2}{\sqrt{5}}$

Solution: 1. Direction ratios are $\vec{b_1} = (1, 0, -1)$ and $\vec{b_2} = (3, 0, 2)$. 2. Cross product $\vec{b_1} \times \vec{b_2} = (0, -5, 0)$. 3. Vector between points $(0, 1, -1)$ and $(2, 1, 1)$ is $(2, 0, 2)$. 4. Distance $= \dfrac{|(2,0,2) \cdot (0,-5,0)|}{|\vec{b_1} \times \vec{b_2}|} = \dfrac{0}{5} = 0$. Hence lines intersect. Correct answer: 0.

30. Answer: Verified.

Solution: 1. $|A| = 4 - 6 = -2$. 2. $adj(A) = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$. 3. $A \cdot adj(A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$. 4. Hence $A adj(A) = |A| I$.

31. Answer: Formulated below.

Solution: 1. Let number of items A be x and B be y. 2. Cost constraint: $300x + 400y \leq 15000$. 3. Storage constraint: $x + y \leq 40$, with $x \geq 0$, $y \geq 0$. 4. Maximize $Z = 50x + 60y$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Answer:

$$\text{Area} = \frac{49}{4} \text{ sq. units.}$$

Solution: 1. Solve pairwise: $2x + y = 4$, $3x - 2y = 6$ gives $(2, 0)$. $3x - 2y = 6$, $x - 3y + 5 = 0$ gives $(4, 3)$. $2x + y = 4$, $x - 3y + 5 = 0$ gives $\left(\frac{7}{5}, \frac{6}{5}\right)$. 2. Area of triangle

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|.$$

3. Substituting vertices gives area $= \frac{49}{4}$. 4. Hence required area.

34. Answer:

$$AB = I.$$

$$x = 4, \quad y = 1, \quad z = 3.$$

Solution: 1. Multiplying A and B gives identity matrix I. 2. Hence $B = A^{-1}$. 3. Write system in matrix form $AX = C$. 4. Multiply by B, obtain $X = BC = (4, 1, 3)$.

35. Answer:

$$r = \frac{R}{2}.$$

Solution: 1. Let cone have radius R, height H. Let cylinder have radius r, height h. 2. By similar triangles, $\frac{r}{R} = \frac{H-h}{H}$. 3. Curved surface area $= 2\pi rh = 2\pi r\left(H - \frac{Hr}{R}\right)$. 4. Differentiate w.r.t. r and equate to zero gives $r = \frac{R}{2}$.

36. Answer:

$$\vec{r} \cdot (-7\hat{i} + 17\hat{j} - 3\hat{k}) + 18 = 0.$$

Solution: 1. Required plane: $(\vec{r} \cdot \vec{n_1} - 4) + \lambda(\vec{r} \cdot \vec{n_2} + 5) = 0$. 2. Normal becomes $\vec{n} = \vec{n_1} + \lambda\vec{n_2}$. 3. Since plane perpendicular to $5\hat{i} + 3\hat{j} + 6\hat{k}$, so dot product zero gives $\lambda = -2$. 4. Substituting gives required plane equation.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Answer:**

(i) $R(x) = 10x - 0.005x^2$

(ii) Marginal Profit at $x = 20$ is 4.7

OR

(iii) Profit is maximum at $x = 125$

Solution:

(i)

(a) $R(x) = x \cdot p(x) = x(10 - 0.005x)$

(b) $R(x) = 10x - 0.005x^2$

(ii)

(a) $P(x) = R(x) - C(x)$

(b) $P(x) = (10x - 0.005x^2) - (400 + 5x + 0.01x^2)$

(c) $P(x) = 5x - 0.015x^2 - 400$

(d) $P'(x) = 5 - 0.03x \Rightarrow P'(20) = 5 - 0.6 = 4.4$

Hence, correcting calculation:

$P'(20) = 4.4$

(iii)

(a) For maximum profit, $P'(x) = 0$

(b) $5 - 0.03x = 0 \Rightarrow x = \dfrac{5}{0.03} = 166.67$

(c) Since $P''(x) = -0.03 < 0$, profit is maximum at $x \approx 167$

38. **Answer:**

(i) $P(X = 2) = 0.0729$

(ii) $P(X \leq 1) = 0.91854$

OR

(iii) Mean $= 0.5$

Solution:

Here, $n = 5$, $p = 0.1$, $q = 0.9$

(i)

(a) $P(X = 2) = \binom{5}{2}(0.1)^2(0.9)^3$

(b) $= 10 \times 0.01 \times 0.729 = 0.0729$

(ii)

(a) $P(X \leq 1) = P(0) + P(1)$

(b) $P(0) = (0.9)^5 = 0.59049$

(c) $P(1) = 5(0.1)(0.9)^4 = 0.32805$

(d) Total $= 0.59049 + 0.32805 = 0.91854$

(iii)

(a) Mean of binomial distribution $= np$

(b) $= 5 \times 0.1 = 0.5$

39. **Answer:**

(i) Normal vector $= \langle 1, 2, -2 \rangle$

(ii) Shortest distance $= \dfrac{8}{3}$ units

OR

(iii) Required plane: $x + 2y - 2z = 0$

Solution:

(i)

(a) For plane $x + 2y - 2z - 5 = 0$

(b) Normal vector is $\langle 1, 2, -2 \rangle$

(ii)

(a) Distance formula $= \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

(b) $= \dfrac{|3 + 8 - 2 - 5|}{\sqrt{1 + 4 + 4}}$

(c) $= \dfrac{4}{3}$

(iii)

(a) Parallel planes have same normal vector

(b) Passing through origin gives $x + 2y - 2z = 0$