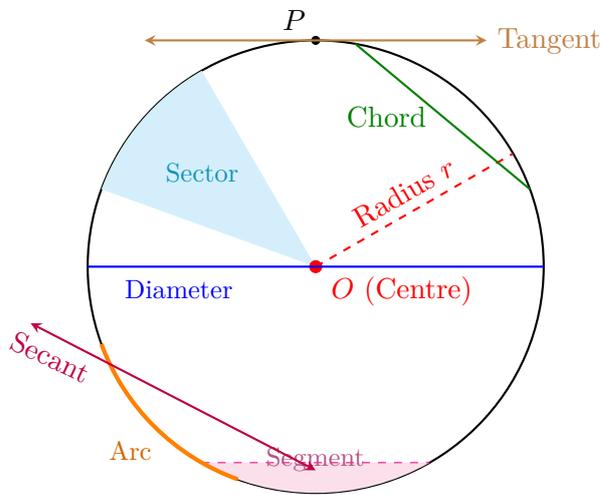


Chapter 10: Circles

This chapter explores the properties of circles, chords, arcs, and cyclic quadrilaterals. You'll learn important relationships between chords, angles, and distances from the center.



1. Key Definitions

- **Radius:** Distance from centre to any point on circle.
- **Diameter:** Longest chord passing through centre; diameter = $2 \times$ radius.
- **Chord:** Line segment joining any two points on circle.
- **Arc:** Part of circumference between two points.
- **Secant:** Line intersecting circle at two points.
- **Tangent:** Line touching circle at exactly one point.
- **Sector:** Region between two radii and arc.
- **Segment:** Region between chord and arc.

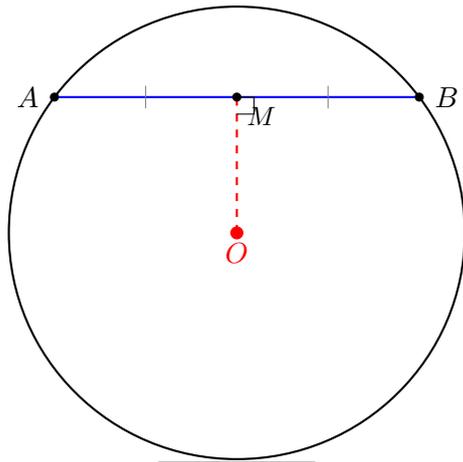
2. Chord Properties

These theorems describe relationships involving chords and the circle's centre.

2. Perpendicular from Centre to Chord

$$\text{If } OM \perp AB \text{ then } AM = MB$$

- O : Centre of circle.
- AB : Chord.
- M : Point where perpendicular from O meets chord.
- **Meaning:** The perpendicular from centre to a chord bisects the chord.
- **Usage:** To find chord length or distance from centre when other is known.



$$OM \perp AB$$

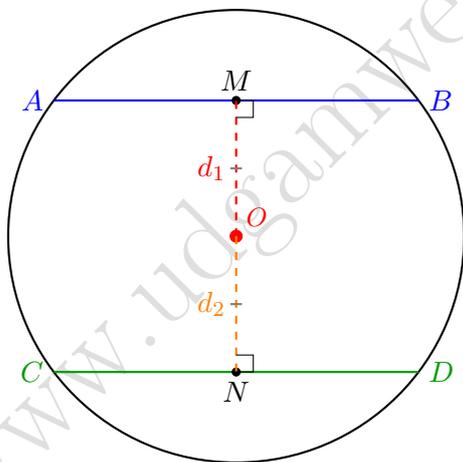
$$AM = MB$$

3. Equal Chords and Distance from Centre

If $AB = CD$ then $OM = ON$

If $OM = ON$ then $AB = CD$

- AB, CD : Two chords.
- OM, ON : Perpendicular distances from centre to chords.
- **Meaning:** Equal chords are equidistant from centre, and conversely, chords equidistant from centre are equal.
- **Usage:** To prove chords equal or to find distances.



$$\text{If } AB = CD \iff OM = ON$$

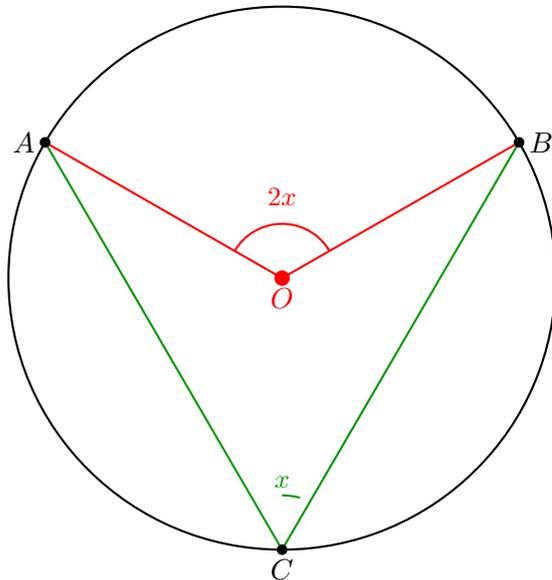
4. Angle Subtended by a Chord at Centre

Angle at centre = $2 \times$ Angle at circumference

$$\angle AOB = 2 \times \angle ACB$$

- AB : Chord.

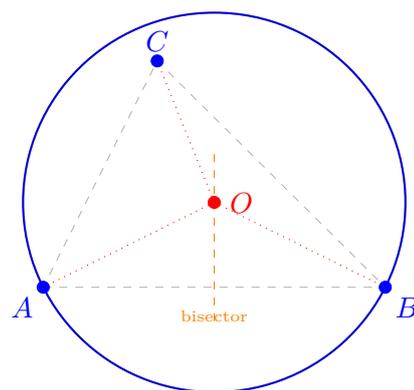
- O : Centre.
- C : Point on circle (on same arc AB).
- **Meaning:** The angle subtended by a chord at centre is double the angle subtended at any point on remaining circle.
- **Usage:** To find unknown angles in circle geometry.



$$\angle AOB = 2 \times \angle ACB$$

3. Circle Through Three Points

This theorem establishes when a unique circle can be drawn through given points.



5. Circle Through Three Non-Collinear Points

One Unique Circle through A, B, C

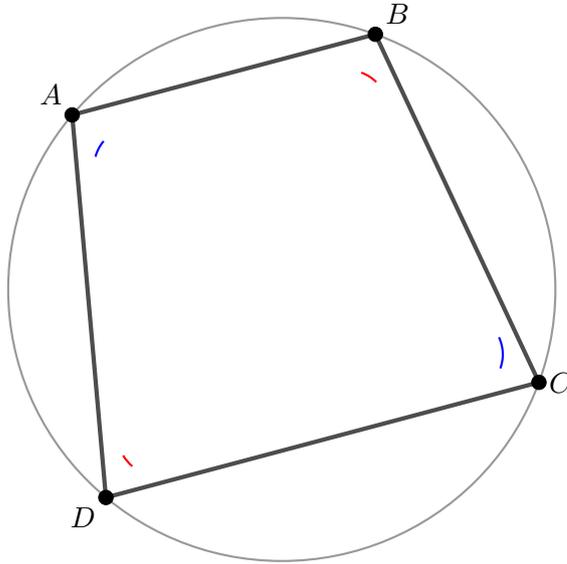
4. Cyclic Quadrilaterals

A quadrilateral whose all vertices lie on a circle is called cyclic.

6. Opposite Angles of Cyclic Quadrilateral

$$\angle A + \angle C = 180^\circ \quad \text{and} \quad \angle B + \angle D = 180^\circ$$

- $\angle A, \angle B, \angle C, \angle D$: Interior angles of cyclic quadrilateral.
- **Meaning:** Opposite angles of cyclic quadrilateral are supplementary (add to 180°).
- **Usage:** To find unknown angles or verify if quadrilateral is cyclic.

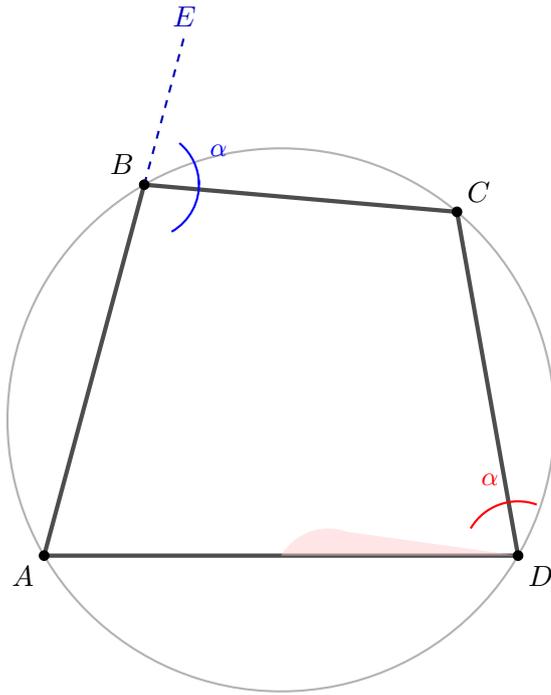


$$\begin{aligned} \angle A + \angle C &= 180^\circ \\ \angle B + \angle D &= 180^\circ \end{aligned}$$

7. Exterior Angle of Cyclic Quadrilateral

$$\angle CBE = \angle ADC$$

- $\angle CBE$: Exterior angle formed by extending side CB .
- $\angle ADC$: Interior opposite angle.
- **Meaning:** Exterior angle equals interior opposite angle in cyclic quadrilateral.
- **Usage:** Useful in proving theorems and solving problems.



Exterior $\angle CBE =$ Interior Opposite $\angle ADC$

5. Important Circle Theorems

8. Angles in Same Segment

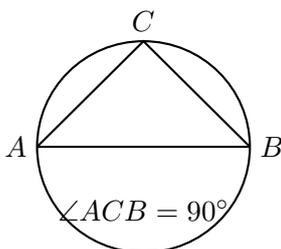
$$\angle ACB = \angle ADB$$

- AB : Chord.
- C, D : Points on the same segment of the circle.
- **Meaning:** Angles in the same segment of a circle are equal.
- **Usage:** To find equal angles in circle-related problems.

9. Angle in Semicircle

$$\angle ACB = 90^\circ$$

- AB : Diameter of the circle.
- C : Any point on the circle.
- **Meaning:** The angle in a semicircle is always a right angle.
- **Usage:** To identify right-angled triangles or prove a triangle is right-angled.



Quick Revision Summary

Here are all the essential theorems and formulas from this chapter.

1. **Perpendicular from centre:** $OM \perp AB \Rightarrow AM = MB$
2. **Equal chords:** $AB = CD \Leftrightarrow OM = ON$
3. **Angle at centre:** $\angle AOB = 2 \times \angle ACB$
4. **Circle through three points:** One unique circle through three non-collinear points
5. **Cyclic quadrilateral:** $\angle A + \angle C = 180^\circ$, $\angle B + \angle D = 180^\circ$
6. **Exterior angle of cyclic quadrilateral:** $\angle CBE = \angle ADC$
7. **Angles in same segment:** $\angle ACB = \angle ADB$
8. **Angle in semicircle:** $\angle ACB = 90^\circ$ (when AB is diameter)